Microeconomic Theory Qualifying Exam

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Instructions. You have four hours to complete this exam, plus a 15 minute "grace period" to wrap up if needed. Answer all four questions. The questions are equally weighted.

Write on **one side** of the provided paper only. Start the answer to each question on a new sheet of paper and be sure to write your candidate number, question number, and page number on each sheet.

Be concise in your answers, and think before you write. Good luck!

1. For the statements in quotation marks in each of parts (a), (b) and (c), indicate whether the statement is True or False, AND justify your answer fully in either case.

(a) "Even if all consumers have the same convex preferences, there may not exist an efficient allocation where everyone receives the same consumption bundle."

(b) "Let (x^*, p^*) be a competitive equilibrium for an exchange economy where all preferences are locally nonsatiated. Now modify the economy by adding a production possibility set Y, containing the origin, which satisfies

$$\max_{y \in Y} p^* \cdot y = 0.$$

That is, the new technology generates maximum profit equal to zero at the prices p^*. Then it is possible that everyone can be weakly better off (and someone strictly better off) in the new economy than in the old economy with allocation x^*."

(c) Let $S = \{1, 2, 3\}$ be the set of possible states of the world and suppose that each state occurs with equal probability. Fix real numbers $0 < a < b < c$. Let $X = (a, b, c)$ be a random variable on S describing the payoff to a security (here a is the payoff in state 1 and so on). Similarly, a second security has payoff vector $Y = (a, c, a)$. Denote by F_X and F_Y the cumulative distribution functions corresponding to X and Y respectively.

"F_X dominates F_Y by first-order stochastic dominance (prove or disprove this). Moreover, as a result, there *must* exist an arbitrage if security X has a lower price than does security Y."

2. A firm and a worker interact over two periods. There are two types of worker, h and ℓ, where the h type produces \$100 for the firm if hired and the ℓ type produces \$10 for the firm if hired. The firm's prior is that the probability of type h is $p \in (0, 1)$ where $100p + 10(1 - p) > 15$. The worker knows his type. The firm and worker are both risk neutral. The worker maximizes his total wages over the two periods while the firm maximizes its total profit over the two periods. The sequence of events:

Period 1.

• The firm offers a contract consisting of a probability of being hired, α, and a wage w conditional on being hired.

• The worker accepts or rejects the offer.

If he rejects, the firm gets 0 and the worker gets his reservation wage of \$15. We then move to the next period.

If he accepts, then with probability α, the worker is hired. In this case, the firm receives an amount of money equal to the worker's productivity and pays the worker w. With probability $1 - \alpha$, the worker is not hired and he and the firm both receive 0 for the period. Either way, we then move to the next period.

Period 2.

• If the worker accepted and was hired in the first period, his productivity is now known to the market. He receives the larger of his productivity or reservation wage, while the firm receives 0. Thus the h type earns \$100 in the second period while the ℓ type earns \$15.

• If the worker did not accept or accepted but was not hired in the first period, his only option in the second period is to get the reservation wage of \$15. The firm gets 0.

(a) Suppose the firm makes a contract offer that both the h and ℓ types accept. What is its best such offer? What profits does it earn from this?

(b) Suppose the firm makes an offer which only the h type accepts. Assuming that the firm can set *any* w, including a negative one, what is the best such offer? What profits does the firm earn in this case? Should the firm offer this contract or the one computed in part (a)?

3. Let $\succeq$ be a preference order over lotteries over the finite set X where $X \subset \mathbf{R}$. Recall the following definitions:

Definition 1. $\succeq$ *satisfies the* independence axiom *if* $p \succeq q$ *implies* $\lambda p + (1-\lambda)r \succeq \lambda q + (1-\lambda)r$ *for all lotteries* r *and all* $\lambda \in [0,1]$.

Definition 2. $\succeq$ *satisfies the* Archimedean axiom *if whenever we have* $p \succ q \succ r$*, there exist numbers* $\alpha, \beta \in (0,1)$ *such that* $\alpha p + (1-\alpha)r \succ q \succ \beta p + (1-\beta)r$.

Suppose $\succeq$ is defined as follows. We have $p \succ q$ if

$$\sum_{x \in X} p(x)x > \sum_{x \in X} q(x)x$$

or if $\sum_{x \in X} p(x)x = \sum_{x \in X} q(x)x$ and

$$\sum_{x \in X} p(x)x^2 < \sum_{x \in X} q(x)x^2.$$

Finally, if $\sum_{x \in X} p(x)x = \sum_{x \in X} q(x)x$ and $\sum_{x \in X} p(x)x^2 = \sum_{x \in X} q(x)x^2$, then $p \sim q$.

Prove your answers to each of the following. (Hint: If you find it helpful, assume $X = \{-1, 0, 1\}$.)

(a) Is $\succeq$ complete and transitive?

(b) Does $\succeq$ satisfy the independence axiom?

(c) Does $\succeq$ satisfy the Archimedean axiom?

4. Consider an election with 3 candidates: A, B and C. The electoral system is plurality. That is, each voter must vote for one of the three candidates (abstention is not allowed) and the candidate that obtains the largest number of votes wins. Ties are broken equiprobably (i.e., 1/2–1/2 in case of a two–way tie and 1/3–1/3–1/3 in case of a three–way tie). Voters only care about the identity of the winner (i.e. voters **do not** get utility directly from voting for a particular candidate).

There are three types of voters: t_A, t_B and t_C. Types t_A prefer A to B to C, types t_B prefer B to A to C, and types t_C prefer C to A and B and are indifferent between A and B. More specifically,

$$U(A|t_A) = 2 > U(B|t_A) = 0 > U(C|t_A) = -1$$
$$U(B|t_B) = 2 > U(A|t_B) = 0 > U(C|t_B) = -1$$
$$U(C|t_C) = 2 > U(A|t_C) = U(B|t_C) = 0$$

where $U(W|t)$ is the utility of a type t voter if the winner of the election is candidate $W \in \{A, B, C\}$.

(a) Assume that there are 3 t_A voters, 3 t_B voters, and 4 t_C voters.

(i) Find all the symmetric Nash equilibria in pure strategies. By "symmetric," I mean that all voters of a particular type play the same strategy.

(ii) Identify the weakly dominated strategies for each type of voters.

(iii) Find all the symmetric Nash equilibria in pure strategies if voters only play weakly undominated strategies.

(b) Assume that there are 6 t_A voters, 4 t_C voters, and no t_B voters. Find all the symmetric Nash equilibria in pure strategies if voters only play weakly undominated strategies.