

Answers to Microeconomic Theory Qualifying Exam

Department of Economics
Boston University

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1. (a) FALSE - assuming smoothness, all MRS's are equal at the equal division allocation.

(b) FALSE - $(x^*, y^* = 0, p^*)$ is a competitive equilibrium in the new economy. By the First Welfare Theorem, the allocation $(x^*, 0)$ is efficient in the new economy.

(c) The FSD ranking is easily verified (in spite of the fact that there is not state-by-state dominance). However, the security market could be arbitrage-free even if $p_X < p_Y$. To prove this, use the Fundamental Theorem. It states that the security market with prices p_X and p_Y is arbitrage-free iff there exist μ_1, μ_2 and μ_3 , positive state prices, such that

$$p_X = a\mu_1 + b\mu_2 + c\mu_3, \quad p_Y = a\mu_1 + c\mu_2 + a\mu_3.$$

If μ_2 is sufficiently large (relative to the other state prices), then $p_X < p_Y$ can be satisfied (because $b < c$), and yet arbitrage is excluded. This may seem counterintuitive given the FSD ranking, but

2. (a) The firm's profits if both types accept is $\alpha[100p + 10(1 - p) - w] = \alpha[100p + 10(1 - p)] - \alpha w$ (note that the firm always gets 0 in the second period). The constraint that h accepts is

$$\alpha[w + 100] + (1 - \alpha)15 \geq 30$$

since accepting gives w today and 100 tomorrow if hired and 15 tomorrow if not, while not accepting gives 15 each period. Similarly, the constraint that ℓ accepts is

$$\alpha[w + 15] + (1 - \alpha)15 \geq 30$$

since if he accepts and is hired today, he'll get his outside option of 15 tomorrow. Obviously, the latter implies the former, so we can disregard the former. Also, this constraint must bind or else the firm could reduce w and increase profits. So the only constraint is $\alpha w = 15$.

Substituting into the profit function gives $\alpha[100p + 10(1 - p)] - 15$. This is obviously increasing in α , so the solution is $\alpha = 1$. Hence $w = 15$. The firm's profits are $100p + 10(1 - p) - 15 = 90p - 5$.

(b) Since the ℓ type is not hired, the profit function is now $\alpha p[100 - w] = p[\alpha 100 - \alpha w]$.

Now we ignore the second constraint above and only have the first one. As before, this must bind, so we get

$$\alpha[w + 85] = 15$$

so $\alpha w = 15 - 85\alpha$. Substituting into the profit function gives

$$p[\alpha 100 - (15 - 85\alpha)] = p[185\alpha - 15].$$

Again, this is increasing in α , so the solution is $\alpha = 1$. Hence $w = 15 - 85 = -70$. Intuitively, the worker pays the firm in order to make his productivity known to the market.

The firm's profits are $p[185 - 15] = 170p$. Thus this contract is better than the one in (a) iff

$$170p \geq 90p - 5$$

which must hold.

3. First, some short-hand: Let $u_1(p) = \sum_{x \in X} p(x)x$ and let $u_2(p) = -\sum_{x \in X} p(x)x^2$. So $p \succ q$ if $u_1(p) > u_1(q)$ or if $u_1(p) = u_1(q)$ and $u_2(p) > u_2(q)$. If $u_i(p) = u_i(q)$, $i = 1, 2$, then $p \sim q$.

(a) To see that \succeq is complete, note that we must have $p \succ q$ or $q \succ p$ if $u_1(p) \neq u_1(q)$. So suppose $u_1(p) = u_1(q)$. Then we must have $p \succ q$ or $q \succ p$ if $u_2(p) \neq u_2(q)$. So the only possibility left is $u_i(p) = u_i(q)$, $i = 1, 2$, in which case $p \sim q$. Hence \succeq is complete.

To see that it is transitive, suppose $p \succeq q$ and $q \succeq r$. Then we must have $u_1(p) \geq u_1(q)$ and $u_1(q) \geq u_1(r)$, so $u_1(p) \geq u_1(r)$. If either of the first two inequalities is strict, then the last one must be, in which case $p \succ r$. So suppose neither is strict — that is, that $u_1(p) = u_1(q) = u_1(r)$. Then we must have $u_2(p) \geq u_2(q) \geq u_2(r)$, so $u_2(p) \geq u_2(r)$. Again, if either of the first two inequalities is strict, the last must be, so $p \succ r$. Finally, if neither is strict, we have $u_i(p) = u_i(r)$, $i = 1, 2$, so $p \sim r$. In all cases, then, $p \succeq r$, so \succeq is transitive.

(b) Suppose $p \succeq q$. Then we must have $u_1(p) \geq u_1(q)$. Hence $\lambda u_1(p) + (1 - \lambda)u_1(r) \geq \lambda u_1(q) + (1 - \lambda)u_1(r)$ for any r and any $\lambda \in (0, 1]$. (For the case of $\lambda = 0$, the conclusion of the independence axiom holds trivially, so I ignore this case throughout.) From the definition of u_1 , though,

$$\lambda u_1(p) + (1 - \lambda)u_1(r) = u_1(\lambda p + (1 - \lambda)r).$$

If $u_1(p) > u_1(q)$, then $u_1(\lambda p + (1 - \lambda)r) > u_1(\lambda q + (1 - \lambda)r)$, implying $\lambda p + (1 - \lambda)r \succ \lambda q + (1 - \lambda)r$ for all r and all $\lambda \in (0, 1]$.

So suppose $u_1(p) = u_1(q)$, implying $u_1(\lambda p + (1 - \lambda)r) = u_1(\lambda q + (1 - \lambda)r)$. Since $p \succeq q$, we must have $u_2(p) \geq u_2(q)$. Repeating the same argument as above with u_2 in place of u_1 shows that if $u_2(p) > u_2(r)$, then we have $\lambda p + (1 - \lambda)r \succ \lambda q + (1 - \lambda)r$ for all r and all $\lambda \in (0, 1]$.

So finally suppose $u_i(p) = u_i(q)$, $i = 1, 2$. Then we have $u_i(\lambda p + (1 - \lambda)r) = u_i(\lambda q + (1 - \lambda)r)$, $i = 1, 2$, implying $\lambda p + (1 - \lambda)r \sim \lambda q + (1 - \lambda)r$.

Thus $p \succeq q$ implies $\lambda p + (1 - \lambda)r \succeq \lambda q + (1 - \lambda)r$ for all r and all $\lambda \in (0, 1]$. Hence the independence axiom holds.

(c) The Archimedean axiom does not hold. To see this concretely, take $X = \{-1, 0, 1\}$. Let $p = (0, 0, 1)$ (i.e., probability 1 on 1), $q = (0, 1, 0)$ (that is, probability 1 on 0), and $r = (1/2, 0, 1/2)$ (probability 1/2 each on -1 and 1). Then $u_1(p) > u_1(q) = u_1(r)$ and $u_2(q) > u_2(r)$. So $p \succ q \succ r$. However, for any $\beta \in (0, 1)$, we have $u_1(\beta p + (1 - \beta)r) > u_1(q)$, so $\beta p + (1 - \beta)r \succ q$.

4. (a)

(i) I denote by $\sigma_t(\psi)$ the probability that a type- t voter plays action $\psi \in \{A, B, C\}$. There are 5 symmetric NE in pure strategies:

$$\begin{aligned}\sigma_t(A) &= 1 \quad \forall t \in \{t_A, t_B, t_C\}, \\ \sigma_t(B) &= 1 \quad \forall t \in \{t_A, t_B, t_C\}, \\ \sigma_t(C) &= 1 \quad \forall t \in \{t_A, t_B, t_C\}, \\ \sigma_t(A) &= 1 \quad \forall t \in \{t_A, t_B\} \text{ and } \sigma_{t_C}(C) = 1, \\ \sigma_t(B) &= 1 \quad \forall t \in \{t_A, t_B\} \text{ and } \sigma_{t_C}(C) = 1.\end{aligned}$$

Importantly, $\sigma_t(A) = 1 = \sigma_t(B) = \sigma_{t_C}(C) = 1$ is not an equilibrium since types- t_A and types- t_B want to deviate.

(ii) For types- t_A and t_B , C is a weakly dominated strategy. For types- t_C , both A and B are weakly dominated strategies.

(iii) There are 2 symmetric NE in pure strategies if voters only play weakly undominated strategies:

$$\begin{aligned}\sigma_t(A) &= 1 \quad \forall t \in \{t_A, t_B\} \text{ and } \sigma_{t_C}(C) = 1, \\ \sigma_t(B) &= 1 \quad \forall t \in \{t_A, t_B\} \text{ and } \sigma_{t_C}(C) = 1.\end{aligned}$$

(b) There are 2 symmetric NE in pure strategies if voters only play weakly undomi-

nated strategies:

$$\begin{aligned}\sigma_{t_A}(A) &= 1 = \sigma_{t_C}(C), \\ \sigma_{t_A}(B) &= 1 = \sigma_{t_C}(C).\end{aligned}$$