Department of Economics Boston University

SOLUTIONS for **Microeconomic Theory Qualifying Exam**

June 2011

1. (a) Adapt the proof of EU thm (b) No and no, because of $\log 0$ (c) For any given (P, α), the agent would want full coverage conditional on participating, since marginal cost of coverage is 0. So for any given level of profit, the efficient contract will set $\alpha = 1$. The company participates if the expected profit is nonnegative and the agent participates if the expected utility of doing so is higher than not doing so. The agent would want to pay smallest P the firm will accept whereas company will want highest P for which agent participates. That is, the smallest P will satisfy $P - [0.2 \times 50000] = 0$ and the largest P will satisfy $\sqrt{80000 - P} = 0.2\sqrt{80000} + 0.8\sqrt{30000}$.

2. (i) A PBE is: (a) a strategy of each worker of type θ_i which is whether or not to submit to the test ($I_i = 1$ or 0); (b) (common) beliefs of employers $\mu_j(I, \theta_i)$ which is a posterior belief that the worker is of type θ_j after observing either $(I = 1, \theta_i)$ for a worker of true type θ_i, or $I = 0$; and (c) a wage offer w_k made thereafter by firm k. It satisfies the requirement that:

(B) the posterior belief is derived from the workers strategy upon applying Bayes rule, which implies that $\mu_j(1, \theta_i) = 1$ if and only if $i = j$, and $\mu_j(0) = \frac{\alpha_j}{\sum_{i:I_i=0} \alpha_i}$ for any j such that $I_j = 0$ (assuming such a j exists), and 0 otherwise. If every type i selects $I_i = 1$ then the beliefs $\mu_j(0)$ consequent on $I = 0$ are arbitrary.

(S) the strategies are sequentially rational given the beliefs, which means for every firm k offers the following wage function : $w(1, \theta_i) = \theta_i$, and $w(0) = E[\theta | I_i = 0]$, where this expectation is formed on the basis of the posterior beliefs, and for any worker of type θ_i: $I_i = 1$ if $\theta_i > w(0)$, $I_i = 0$ if $\theta_i < w(0)$, and I_i either 0 or 1 if $\theta_i = w(0)$.

(ii) There is a PBE in which every type of worker decides to submit to the test, combined with pessimistic beliefs ($\mu_1(0) = 1$) held by firms consequent upon observing a worker that does not take the test. Then $w(0) = \theta_1$ so every worker of type θ_2 or higher strictly prefers to take the test, while workers of type θ_1 are indifferent.

(iii): Consider any PBE where some type θ_i decides not to submit to the test: by (S) we have $\theta_i \leq w(0) = E[\theta | I_i = 0]$ for any such type. It follows that only one such type must not take the test (otherwise, take the highest type that does not take the test, this condition must be violated for that type). Moreover since for any other type θ_j we have $I = 1$ we must have $\theta_j \geq w(0) = E[\theta | I_i = 0]$, so the only type that does not take the test must be type θ_1. Then all types must reveal themselves: all types θ_2 and above take the test, and type θ_1 reveals herself by not taking the test.

3. (i) The order of deletion is not crucial for strict dominance, so the following one order. First, x strictly dominates z for 1. Once z is eliminated, b strictly dominates both a and d for 2. After eliminating these two strategies, both x and y strictly dominate w for 1. No further elimination is possible (with pure or mixed strategies), so what survives is x and y for 1, b and c for 2.

(ii) Order is critical for weak dominance, so what follows is *necessarily* the order in which elimination must take place. On the first round, x dominates z for 1 (just as it did before) and 1 has no other dominated strategies. For 2, a is dominated by b and 2 has no other dominated strategies. Hence we eliminate z and a on the first round. On the second round, no strategies are dominated for 1, but both of 2's pure strategies dominate d. Hence only d is eliminated on the second round. On the third round, y dominates both of 1's other two pure strategies, but 2 has no dominated strategies. Hence we eliminate only w and x. Finally, on the fourth round, we eliminate b for 2, leaving only (y, c).

(iii) The answer is the same as for (i). Any strategy that is strictly dominated cannot be a best reply; any strategy that is not strictly dominated by a mixed strategy is a best reply (which is easy to verify in the specific cases here).

(iv) A strategy which is not rationalizable must have zero probability in any Nash equilibrium. Hence for this part and the next, we can restrict attention to the matrix

	b	c
x	$2,4$	$3,4$
y	$2,3$	$4,5$

The pure equilibria are (x, b) and (y, c).

(v) There are no mixed equilibria other than the pure equilibria from (iv). To see this, suppose we have an equilibrium in which 1 uses a mixed strategy which puts strictly positive probability on both x and y. Since c dominates b, 2's best reply to any such mixture is c. Hence if there is such an equilibrium, 2 plays c. But 1's best reply to this is y, a contradiction. Similarly, suppose there is an equilibrium in which 2 uses a mixed strategy which puts strictly positive probability on both b and c. Then 1 must play y as it is his best response. But 2's best response to y is c with probability 1, a contradiction.

4. (a) The statement is false. The intuition is that the *gains from trade* starting at ω_1' can be much smaller than those starting from ω_1. For example, let consumer 2 have linear indifference curves corresponding to $u_2(x_2, y_2) = 10x_2 + y_2$. Then any interior equilibrium will have relative prices equal to $p_2/p_1 = 10$. Suppose that ω_1 has much more of good x than of y, and that the converse is true for ω_1'. One can construct a counterexample to the assertion along these lines.

[35% total. 25% for correct intuition, 10% for example sketch]

(bi) Let $(\overline{\omega}_0, (\overline{\omega}_{1s})_{s=1}^S)$ be the initial endowment of the representative agent. Then p is an equilibrium price for the rep agent economy iff, for every s,

$$p_{1s} = \frac{(\beta^* \pi_s / \overline{\omega}_{1s})}{(1/\overline{\omega}_0)} = (\pi_s / \overline{\omega}_{1s}) (\Sigma_i \beta_i x_{i0}^*) \tag{1}$$

Thus we need only show that this equation is implied by the hypothesis that (x^*, p) is an Arrow-Debreu equilibrium for the heterogeneous agent economy. Equate MRS to price for consumer i to obtain

$$\begin{aligned} p_{1s} &= \frac{(\beta_i \pi_s / x_{i1s}^*)}{(1/x_{i0}^*)} = (\beta_i x_{i0}^* \pi_s / x_{i1s}^*) \implies \\ p_{1s} x_{i1s}^* &= \beta_i \pi_s x_{i0}^* \implies \\ \Sigma_i p_{1s} x_{i1s}^* &= p_{1s} \overline{\omega}_{1s} = \pi_s (\Sigma_i \beta_i x_{i0}^*), \end{aligned}$$

which proves (1).

[40% total. Small penalty for calculation error.]

(bii) The representative agent's discount factor, and hence preference, depends on the initial equlibrium. Thus the representative agent varies with the equilibrium being replicated. Consequently, it is in general useless for comparative statics or welfare questions.

[25%]