Department of Economics Boston University

Microeconomic Theory Qualifying Exam
June 2011

Instructions. You have four hours to complete this exam, plus a 15 minute "grace period" to wrap up if needed. Answer all four questions. Questions are equally weighted.

Write on **one side** of the provided paper only. Start the answer to each question on a new sheet of paper and be sure to write your candidate number, question number, and page number on each sheet.

Be concise in your answers, and think before you write. Good luck!

1. (a) Suppose that an agent is an expected utility maximizer. Outline a choice-based procedure by which we can elicit her utility for money. Assume that we are interested in money amounts between \$0 and some maximum $\$M > 0$.

(b) Consider an agent whose preference $\succsim$ over lotteries over $\mathbb{R}_+$ is represented by the function $EU(p) = \int \log x \; dp(x)$. Does $\succsim$ satisfy (i) Continuity, (ii) Independence? Provide a proof or counterexample in each case.

(c) Consider an expected utility maximizer with vNM utility index $u(x) = \sqrt{x}$ and wealth $80,000$ who faces a loss of $50,000$ with probability 0.2. A risk neutral insurance company offers policies that take the form of pairs (P, α) where P is a flat premium and α is the fraction of the loss covered.

(i) Characterize the set E of contracts that are efficient and acceptable to both parties.

(ii) Find the contract in E that the agent prefers the most. Find the contract in E that the company prefers the most. Interpret.

2. A labor market has workers whose productivity θ is distributed over a set of n possible values $\{\theta_1, \ldots, \theta_n\}$ with frequencies $\alpha_1, \ldots, \alpha_n$, where $\theta_i < \theta_{i+1}$ and $\alpha_i > 0, \sum_i \alpha_i = 1$. Each worker is privately informed about her own productivity. Each worker decides whether or not to submit to a (costless) test which if taken publicly reveals her productivity without error. All employers (there are at least two) have a constant-returns-to-scale technology and so can hire as many workers as they like. They observe whether any given worker submitted to the test, and if so the result of the test. Then they make wage offers simultaneously to each worker. All workers have a zero reservation wage.

(i) Define a Perfect Bayesian Equilibrium (PBE) in pure strategies for this market game.

(ii) Construct a PBE.

(iii) Show that all PBEs will result in the same allocation.

3. Consider the following game:

		2			
		a	b	c	d
	w	$1, 2$	$1, 3$	$2, 1$	$5, 1$
1	x	$4, 2$	$2, 4$	$3, 4$	$4, 3$
	y	$3, 2$	$2, 3$	$4, 5$	$2, 2$
	z	$3, 2$	$1, 2$	$2, 2$	$1, 4$

Note: In the first two parts below, show the order of eliminations and explicitly mention the strategies (pure or mixed) that dominate the strategies being eliminated.

(i) What strategies survive iterative deletion of strictly dominated strategies? (Check for strict dominance by mixed strategies.)

(ii) What strategies survive iterative deletion of weakly dominated strategies? (Check for weak dominance by mixed strategies.)

(iii) What strategies are rationalizable?

(iv) Find all pure strategy Nash equilibria. If none exist, prove it.

(v) Find all mixed strategy Nash equilbria (not including the pure ones above). If no such equilibria exist, prove it.

4. (a) “The private ownership exchange economies $\mathcal{E}'$ and $\mathcal{E}$ are identical except for one sole difference: in $\mathcal{E}'$ the initial endowment of consumer 1 is ω_1' while in $\mathcal{E}$ his endowment is ω_1. It so happens that $\omega_1' \succ_1 \omega_1$. Let x' and x be the unique Walrasian equilibrium allocations for the respective economies. Then $x_1' \succeq_1 x_1$.” Indicate whether this assertion is True or False. In either case, justify your answer and provide an intuitive explanation.

(b) Consider an economy with I consumers, S states of the world and two periods, $t = 0, 1$. A scalar consumption good is available in each period. Consumer i has utility function U_i, where

$$U_i\left(x_{i0}, (x_{i1s})_{s=1}^S\right) = \log x_{i0} + \beta_i \Sigma_s \pi_s \log x_{i1s}.$$

Here: $(x_{i1s})_{s=1}^S$ denotes random consumption in period 1, $0 < \beta_i < 1$ is a discount factor that varies across consumers, and $\pi = (\pi_s)_{s=1}^S$ gives the common prior probability measure over future states. Finally, initial endowments are given by $\omega_i = \left(\omega_{i0}, (\omega_{is})_{s=1}^S\right) >> 0$; aggregate endowments are denoted by

$$\overline{\omega}_0 = \Sigma_i \omega_{i0} \text{ and } \overline{\omega}_{1s} = \Sigma_i \omega_{i1s},\ s = 1, ..., S.$$

Let (x^*, p) denote an Arrow-Debreu equilibrium for this economy, where $p = (p_{1s})_{s=1}^S$ and period 0 consumption is the numeraire. (To clarify the notation, i receives (x_{i0}^*, x_{i1}^*) in equilibrium.)

(i) Show that p is also an equilibrium price for the representative agent economy where the representative agent has the aggregate endowment and the utility function U given by

$$U\left(x_0, (x_{1s})_{s=1}^S\right) = \log x_0 + \beta^* \Sigma_s \pi_s \log x_{1s}, \text{ and}$$

$$\beta^* = \Sigma_i \beta_i \left(\frac{x_{i0}^*}{\Sigma_j \omega_{j0}}\right) = \Sigma_i \beta_i \left(x_{i0}^* / \overline{\omega}_0\right).$$

(ii) Interpret the above expression for β^* and explain its significance for representative agent modeling.