Solutions for Microeconomic Theory Qualifying Exam  
August 2011

1. First, consider separating equilibria. Note that the usual argument that the low type must get zero education doesn’t hold here. If the low type were supposed to get education 0, he would have utility 40. Suppose he deviated to education 1. His payoff must then be at least $80 - 35 = 45 > 40$ since the worst response possible would be if the employers inferred he was the low type. Hence he would deviate. Therefore, the low type must get education 1 in any separating equilibrium.

Clearly, it can’t be true that the high type gets lower education than the low type in some separating equilibrium. If we try to set this up as an equilibrium, the low type would deviate to imitating the high type and be better off. Hence the high type must choose education strictly above 1 in any separating equilibrium.

Letting $e_H$ denote the education level for the high type, we see that incentive compatibility requires $45 \geq 110 - 35e_H$ and $110 - 15e_H \geq 80 - 15 = 65$. Rearranging, we require $e_H \geq 13/7$ and $e_H \leq 3$. For any choice of $e_H \in [13/7, 3]$, we have a separating equilibrium where the beliefs of the employers are (for example) that the worker is type $\theta_H$ iff $e \geq e_H$ and is type $\theta_L$ otherwise.

Now consider pooling equilibria. First, consider a pooling equilibrium at $e^* \in (0, 1)$. In this case, we need to ensure that the worker doesn’t want to deviate to 0 or 1. Obviously, the worst belief in either case is that he is the low type. Hence we require

\[
(1/2)(100 + 40) - 35e^* \geq 40  \\
(1/2)(100 + 40) - 35e^* \geq 45  \\
(1/2)(100 + 40) - 15e^* \geq 80 - 15 = 65.
\]

The first ensures that the low type doesn’t want to deviate to 0, the second that he doesn’t want to deviate to 1, and the third ensures that the high type doesn’t want to deviate to 1. (If the low type doesn’t want to deviate to 0, the high type doesn’t either, so I didn’t write this one down.) Obviously, the second inequality implies the first. So we need $e^* \leq 5/7$ and $e^* \leq 1/3$. The second constraint is the more severe, so the only pooling equilibria with $e^* \in (0, 1)$ have $e^* \leq 1/3$. Again, it is easy to see that for any $e^* \in (0, 1/3]$, we get a pooling equilibrium by having the beliefs be 50-50 if $e \geq e^*$ and probability 1 on $\theta_L$ otherwise. It’s also easy to see that the same argument supports pooling at $e^* = 0$, so we have pooling equilibria for every $e^* \in [0, 1/3]$. 

1
Now consider the case where $e^* = 1$. (I find it a little simpler to separate this case from $e^* > 1$.) In this case, the worker must get paid $(1/2)(110 + 80) = 95$. Again, take the beliefs to be the same as the above. Then this is an equilibrium as long as

$$95 - 15 \geq 40$$
$$95 - 35 \geq 40$$

both of which hold.

Finally, consider the case where $e^* > 1$. Again, take the beliefs to be the same as above. Then this is an equilibrium iff

$$95 - 15e^* \geq \max\{40, 80 - 15\} = 65$$
$$95 - 35e^* \geq \max\{40, 80 - 35\} = 45$$

Hence we require $e^* \leq 2$ and $e^* \leq 10/7$. Since both must hold, we have an equilibrium when $1 < e^* \leq 10/7$.

So the set of pooling equilibria is $0 \leq e^* \leq 1/3$ and $1 \leq e^* \leq 10/7$. 

2
2. Suggested weights out of 25 in brackets.
   a) [4] $d = \frac{1}{2}$, payoff = 0
   b) [5] (The message space is restricted so that one does not need to know what “measurable” means.) Suppose $\rho(\sigma(\theta)) = \theta$ for all $\theta$. Then in any state $\theta_0$, M gains by deviating to the message $m = \sigma(\theta_0 + b)$. Perfect communication (or perfect separation) cannot occur (without commitment by the principal) when interests aren’t perfectly aligned.
   c) [4] (i) $\sigma(0) = \bar{\theta}/2, \sigma(1) = (1 + \bar{\theta})/2$; for off-equilibrium, any partition $(M_1, M_2)$ of the rest of the message space $M\setminus\sigma([0, \bar{\theta}]), \sigma([\bar{\theta}, 1])$ so that $\rho(m) = \bar{\theta}/2$ if $m \in M_1$ and $\rho(m) = (1 + \bar{\theta})/2$ if $m \in M_2$ will do.
   [4] (ii) At $\bar{\theta}$, M must be indifferent between sending the two messages (easy to see he prefers the first interval for $\theta < \bar{\theta}$, the second for $\theta > \bar{\theta}$, else should change $\bar{\theta}$, $(\bar{\theta} + b - \bar{\theta}/2)^2 = (\bar{\theta} + b - (1 + \bar{\theta})/2)^2 \rightarrow \bar{\theta} = 1/2 - 2b$
   d) [4] Need $1 > 1/2 - 2b > 0$. First condition doesn’t bind; second requires $b < 1/4$. Otherwise the message is [0,1] and $d = 1/2$. If H and M are too far apart (bias $b$ too large), no effective communication
   (strictly speaking this is a slight over-interpretation since we haven’t asked them to show this is the "best" equilibrium in this case; in fact the condition $b > 1/12$ ensures it is, and the interpretation generalizes).
   e) [4] If $b < 1/4$, communication is preferred (otherwise H is indifferent). Let $p(0) = \Pr(\theta \leq \bar{\theta}), \pi(d|0)$ be the expected payoff given $\theta \leq \bar{\theta}$ and d is chosen; similarly for $p(1)$ and $\pi(d|1)$. Then $\pi(\frac{\bar{\theta}}{2}|0) > \pi(\frac{1}{2}|0)$. Then $\pi(\frac{1}{2}|0) > \pi(\frac{1}{2}|1)$ by revealed preference and expected communication payoff is $p(0)\pi(\frac{\theta}{2}|0) + p(1)\pi(\frac{1+\theta}{2}|1) > p(0)\pi(\frac{1}{2}|0) + p(1)\pi(\frac{1}{2}|1) = \text{autarky payoff.}$

   Or by direct computation (messy), the loss under communication is $\int_{\bar{\theta}}^{1/2} (\frac{\theta}{2})^2 d\theta + \int_{\bar{\theta}}^{1/2} (\frac{\theta}{2} - \frac{1}{2})^2 d\theta = \int_{\bar{\theta}}^{1/2} (\frac{\theta}{2} - \frac{\theta}{2})^2 d\theta + \int_{\bar{\theta}}^{1/2} (\frac{\theta}{2} - \theta + \frac{1}{4}) d\theta = \frac{1}{12} + \frac{b^2 - \bar{\theta}^2}{4} = \frac{1}{12} - \frac{1}{4}(\frac{1}{2} - 2b)(\frac{1}{2} + 2b)$

   $= \frac{1}{12} - \frac{1}{16} + b^2 < \frac{1}{12}$ (the autarky loss) whenever $b < 1/4$. 

3
3. (i) In the Bayesian case, complete insurance is Pareto optimal iff the consumers have identical beliefs. The generalization to this case where beliefs are represented by intervals is that the two intervals must intersect, which is true iff \( p \leq \frac{1}{2} \). (An Edgeworth box diagrammatic argument would suffice as proof.) The generalization to permit \( u \) to be nonlinear, but concave and differentiable, does not change matters because the set of MRS’s at certainty is unchanged for each consumer: consider \( pu(x_{j1}) + (1 - p) u(x_{j2}) \), for example. Then the set of hyperplanes tangent to a given bundle along the certainty line is identical to the set prevailing when \( u \) is linear [use diagram]. The same is true for the min of two such expected utility functions. This implies that the answers in (i) are still valid.

(ii) The noted assertion presumes expected utility preferences, which is not the case here.

4. (i) Preference is represented by \( u(\cdot) \) and is not affected by price. This is the standard model and Slutsky is satisfied.

(ii) Slutsky is violated in general. One approach is to use brute force. An indirect argument is that Slutsky (plus homogeneity) is sufficient for total household demand to be rationalizable by some utility function \( U(x_A + x_B, y_A + y_B) \). But here the maximand is \( u_A(x_A, y_A) \cdot u_B(x_B, y_B) \), which does not have the preceding form.

(iii) The demand for state contingent consumption is a special case of the standard utility maximization problem, with AD prices replacing the usual commodity prices. Therefore, Slutsky is satisfied. The state dependence feature of preference is irrelevant.