**Kennedy School of Government**
**Harvard University**
**ITF-346 International Financial Policy: Issues and Analysis**
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**Problem Set 1**
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Questions have equal weight: 25 points each. Please answer clearly and show your work. Unreadable answers will be marked down. Use diagrams where useful. Good luck.

## Question 1: Blanchard, Giavazzi and Sa (2005) with endogenous wealth

The idea of this exercise is to study the implications of endogenizing the evolution of wealth, $W$, in Blanchard, Giavazzi and Sa (2005) model, as changes in wealth are an important source of current account dynamics.

(i) Write down the portfolio balance and the current account balance relations and explain them in words. To consider the implications of having wealth

endogenous, include U.S. domestic wealth, $W$, as a determinant of trade deficit. Assume that the derivative of the trade deficit with respect to U.S. wealth is positive, explain the economic intuition of this assumption. (7 points)

(ii) Write down the steady-state equations under the assumption that the expected and realized relative return on U.S. assets are equal $r = r^*$ and $E^e_{+1} = E_{+1}$, thus $R^e = 1$ in steady state. Derive the phase diagram. What are the implications of including wealth as a determinant of trade deficit for the curves as compared to Blanchard, Giavazzi and Sa's analysis? (8 points)

(iii) Consider the effects for the current account and the exchange rate of an unexpected and permanent exogenous increase in the value of U.S. assets, $X$. What are the dynamics assuming that the economy was in equilibrium before the shock? How do the results compare to the experiment of an exogenous shift in demand, $z$, analyzed in the paper? (10 points)

## Question 2: Obstfeld (2004) with external assets

The idea of this exercise is to study the implications of a portfolio model when the inhabitants of the country with original sin have a sizable portfolio of foreign assets. This would be the case of countries such as Lebanon, South Africa or Argentina. Does this fact reduce the "overshooting" properties of the exchange rate adjustment to trade flows? This is what this problem analyses.

i) Take the portfolio model in Obstfeld (2004) but assume that $F$, the net stock of debt, is now positive. Given that it is positive net foreign assets earn the international rate of return $i^*$. Write down the porftolio equilibrium equation for domestic bonds. Derive from it the $\dot{E} = 0$ equation in the $E, F$ space. (5 points)

ii) Assume now that the $\dot{F} = \gamma(\epsilon, F, z)$ with partial derivatives being positive. Explain where the signs of the derivatives come from. Derive the phase diagram in the $E, F$ space. (5 points)

iii) Work through the dynamics to a deteriaration in the trade balance equation, say due to a negative terms of trade shocks (a fall in $z$). Is there overshooting now as in the case seen in class? (5 points)

iv) Work through the dynamics to a sudden stop. Is there overshooting of the exchange rate? (5 points)

v) What would it mean and how would your answers change if $\frac{d\dot{F}}{dF}$ is negative? (5 points)

## Question 3: Taxes in the open economy

Consider the following model of investment. The representative household now owns bonds and firms. The representative firm transfers its profits to the household. The budget constraint of the household is

$$\dot{b}_t = rb_t + \pi_t + \tau_t - c_t \tag{1}$$

where $\tau_t$ are transfers from the government and $\pi_t$ are profits received from the firm. The household take these flows as given when deciding how much to save.

Firms are competitive. The profit function of the representative firm is

$$\pi_t = f(k_t)(1-\gamma_t) - \frac{1}{2\chi}\frac{\left(\dot{k}_t\right)^2}{k_t} - \dot{k}_t \tag{2}$$

where $f(k_t)$ is output and $\frac{1}{2\chi}\frac{\left(\dot{k}_t\right)^2}{k_t}$ is the cost of investing at the rate $\dot{k}_t$, when the stock of capital is $k_t$. Note $\gamma_t < 1$ is the corporate tax rate this company has to pay. Suppose the firm faces the constant market interest rate $r$, and aims to maximize the present discounted value of profits over the infinite future. Time starts at some time 0, when the inherited stock of capital is $k_0 > 0$. Hence, the firm maximizes

$$\int_0^\infty \pi_t e^{-rt} dt \tag{3}$$

taking the whole sequence of taxes as given.

Finally, assume that the government transfers back the revenues from the tax to household:

$$\tau_t = \gamma_t f(k_t) \text{ for all } t \tag{4}$$

a) Solve the household's consumption-savings problem for an arbitrary flow of profits $\pi_t$ and transfers $\tau_t$, for $t$ between zero and infinity. (4 points)

b) Assume $\gamma_t = \gamma^H$ constant for all $t$. Solve the representative firm's investment problem. Characterize the dynamic equations that pin down the evolution of capital and its price. Show also the steady state. How do steady state capital holdings and the price of capital depend on the tax rate? Hint: the Hamiltonian of the firm's problem is

$$H = \pi_t + q_t \dot{k}_t \tag{5}$$

where $q_t$ is the costate associated with the state $k_t$, and $\dot{k}_t$ is the firm's control variable (7 points).

c) Suppose that unexpectedly and forever, the tax rate falls to $\gamma^L < \gamma$. Characterize the short and long run responses of capital and its price. What happens to consumption, savings and the current account? What is the intuition? (7 points)

d) Suppose that, starting at time zero the government announces a temporary (and unexpected) reduction in tax rates:

$$\gamma_t = \begin{Bmatrix} \gamma^L & , 0 \leq t < T \\ \gamma^H & , \ t \geq T \end{Bmatrix} \tag{6}$$

where $\gamma^H > \gamma^L$. Characterie the short run and long run responses of capital and its price. What happens to consumption, savings and the current account? What is the intuition? (7 points)

## Question 4: Imperfect capital markets

Consider the following model of investment. The representative household $n$.

Assume that a country has no government but is populated by a fixed and large number $n$ of exactly identical consumers, indexed by $i$. Each household is exactly equal to the representative consumer we have used in all the models so far. That is, household $i$ maximizes

$$\int_0^\infty \left(\frac{\sigma}{\sigma - 1}\right) \left(c_t^i\right)^{\left(\frac{\sigma-1}{\sigma}\right)} e^{-\delta t} dt \qquad , \sigma > 0 \tag{7}$$

subject to

$$\dot{b}_t^i = r b_t^i + y - c_t^i \tag{8}$$

and

$$\lim_{T\to\infty}\left(b_T^i e^{-rT}\right)\geq 0 \tag{9}$$

Note a superscript $i$ denotes that a variable refers to a single household, and the same variable without superscript refers to the aggregate quantity. You can assume $y$ is constant (unless told otherwise) and the same for all households.

The novelty is that there exists an imperfection in the international capital markets: the interest rate faced by the country is a function of the total amount of debt held by the economy:

$$r_t = r\left(b_t\right) \tag{10}$$

where $r' < 0$, and $r'' < 0$. Note $b_t = \sum_{i=1}^{n} b_t^i$. Note there is a level of $b$ - call it $b^*$- so that $\delta = r\left(b^*\right)$. Suppose, without loss of generality, that $b^* = 0$. Notice this means that $r_t > \delta$ when the economy is a net debtor, and $r_t < \delta$ when the economy is a net creditor vis a vis the rest of the world.

a) Solve the problem for a single household assuming it takes the international rate as given. Abstract from any strategic considerations, i.e. the households do not consider the decisions of the other households in their optimization problem. Each household also takes the aggregate $b_t$ as given when solving its problem, since $n$ is large. Express your solution as a pair of differential equations in $c_t^i$ and $b_t^i$, also including $b_t$ in the term $r\left(b_t\right)$. (5 points)

b) Now solve the problem for the aggregate equilibrium in the economy by summing across all agents and letting $c_t = \sum_{i=1}^{n} c_t^i$, and endogenizing the interest rate paid by the economy as a whole using $r\left(b_t\right)$. Express your solution as a pair of differential equations in $c_t$ and $b_t$. Characterize the steady state of this system. Give intuition as to why the variable take the $SS$ value they take. (5 points)

c) Draw the phase diagram for the system of differential equations in $c_t$ and $b_t$. You can assume the condition $-\frac{b^* r'(b^*)}{r(b^*)} < 1$, which guarantees that the system displays saddle path stability. Use the phase diagram to show the effects of an unanticipated and permanent increase in $y$. Explain what happens to consumption, bond-holdings and the rate of interest. Give intuition. (5 points)

d) Suppose that a central planner takes over the economic decision of the inhabitants of this country. This social planner is very benevolent, so her objective function is equal to the arithmetic average of the utility function of the $n$ agents in the country. State the social planner's optimization problem and be specific about all the restrictions she faces. Characterize the solution to this problem using the tools you know. Again express your solution as a pair of differential equations in $c_t$ and $b_t$, and show their steady state. (5 points)

e) Is the solution to the decentralized economy and the central planner's problem the same? Is the steady state the same? Are dynamics around the steady state the same? Why or why not? Explain the intuition for your results. (5 points)