Experimental observation of self-replicating spots in a reaction-diffusion system

Read the paper and understand the biology and mathematical models constructed by the authors. You are required to make presentation and project report with the following details:

1. Find all possible steady-state points of the (already non-dimensionalized) equations on page 216. [10 points]
2. Linearize the non-dimensional equations by taking into account inhomogeneous perturbations. [10 points]
3. Calculate eigenvalues from the linearized equations (calculate Jacobian matrix and solve the characteristic polynomial). [10 points]
4. Investigate the stability of all steady states. [10 points]
5. Produce dispersion relations from each steady state. [10 points]
6. Solve/simulate the non-dimensional equations in 2D space using either Crank-Nicolson, Alternating Direct Implicit (ADI), or Method of Lines (choose only 1 method):
   - Reproduce figure 1, 2, 3, and 4 in the paper. [30 points]
7. Solve/simulate the non-dimensional equations in 1D space using Forward, Backward, and Crank-Nicolson methods:
   - Compare the results of each method.
   - Determine the most efficient method to use (based on accuracy and time).
   - Using the most efficient method, plot the solutions of all variables that correspond to the 2D solutions in (6).
[20 points]

Presentation and report should include:

- Biological background.
- Mathematical formulation.
- Mathematical analysis (non-dimensionalization and linear stability analysis).
- Numerical methods used and results of simulations.
- Biological interpretation of results and discussion