Modeling the interaction of keratinocytes and fibroblasts during normal and abnormal wound healing processes

Read the paper and understand the biology and mathematical models constructed by the authors. You are required to make presentation and project report with the following details:

(1) Non-dimensionalize equations (2.1), (2.2), (2.3), (2.4), (2.5), (2.6), (2.7), and (2.8).
[5 points]

(2) Find all possible steady-state points of the non-dimensional equations.
[5 points]

(3) Linearize the non-dimensional equations by taking into account inhomogeneous perturbations.
[10 points]

(4) Calculate eigenvalues from the linearized equations (calculate Jacobian matrix and solve the characteristic polynomial using Bairstow’s method).
[10 points]

(5) Investigate the stability of all steady states.
[10 points]

(6) Produce dispersion relations from each steady state.
[10 points]

(7) Solve/simulate the non-dimensional equations in 1D space using Forward, Backward, and Crank-Nicolson methods:
   
   – Compare the results of each method from the solutions of keratinocytes and extracellular matrix.
   – Determine the most efficient method to use (based on accuracy and time).
   – Using the most efficient method, plot the solutions of all variables, equations (2.1) to (2.8).

[20 points]
(8) Solve/simulate the non-dimensional equations in 2D space using either Crank-Nicolson, Alternating Direct Implicit (ADI), or Method of Lines (choose only 1 method):

– Reproduce figure 3 and 4 in the paper.
– Produce 2D solutions for fibroblasts that correspond to solutions in figures 3 and 4 for keratinocytes.
– Produce 2D solutions for other variables/components of the model.

[30 points]

Presentation and report should include:

• Biological background.
• Mathematical formulation.
• Mathematical analysis (non-dimensionalization and linear stability analysis).
• Numerical methods used and results of simulations.
• Biological interpretation of results and discussion