Assignment 3 - BE 503/703

Due: November 3rd, 2015

Interacting Ion Channels: The Morris-Lecar Model

The Morris-Lecar model is a biological neuron model developed by Catherine Morris and Harold Lecar to reproduce the variety of oscillatory behavior in relation to Ca\(^+\) and K\(^+\) conductance in the giant barnacle muscle fiber.

Application of a depolarising current to barnacle muscle fibers produces a broad range of electrical activity. Careful experimental work by a number of research groups has indicated that the giant barnacle muscle fiber contains primarily voltage gated K\(^+\) and Ca\(^+\) currents along with a K\(^+\) current that is activated by intracellular Ca\(^+\), a so-called K\(^+_\text{Ca}\). Neither of the voltage gated currents shows significant inactivation in voltage clamp experiments. Morris and Lecar proposed a simple model to explain the observed electrical behaviour of the barnacle muscle fibers [Morris and Lecar, 1981]. Their model involves only a fast activating Ca\(^+\) current, a delayed rectifier K\(^+\) current, and a passive leak. They tested the model against a number of experimental conditions in which the interior of the fibre was perfused with the Ca\(^+\) chelator EGTA in order to reduce activation of the K\(^+_\text{Ca}\) current. Their simulations provide a good explanation of their experimental measurements. The model translates into two equations:

\[
\frac{dV}{dt} = \frac{-g_{\text{Ca}}m_{\infty}(V)(V - V_{\text{Ca}}) - g_K w(V - V_K) - g_L(V - V_L) + I_{\text{app}}}{C_d} \quad (1)
\]

\[
\frac{dw}{dt} = \frac{\phi(w_{\infty}(V) - w)}{\tau(V)} \quad (2)
\]

where \(V\) is membrane potential and \(w\) is fraction of open K\(^+\) channels. Here \(m_{\infty}(V)\) is the fraction of voltage-dependent Ca\(^+\) channel open and this is a function of voltage but not time, \(w_{\infty}(V)\) is the fraction of open channels for the delayed rectifier K\(^+\) channels, and \(\tau(V)\) is associated with the relative time scales of the firing dynamics, which varies broadly from cell to cell and exhibits significant temperature dependency. The conductances \(g_L, g_{\text{Ca}},\) and \(g_K\) are for the leak, Ca\(^+\), and K\(^+\) currents, respectively.
The functions $m_\infty(V)$, $w_\infty(V)$, and $\tau(V)$ are defined as

$$m_\infty(V) = 0.5 \left[ 1 + \tanh \left( \frac{V - v_1}{v_2} \right) \right]$$  \hspace{1cm} (3)$$

$$w_\infty(V) = 0.5 \left[ 1 + \tanh \left( \frac{V - v_3}{v_4} \right) \right]$$  \hspace{1cm} (4)$$

$$\tau(V) = \frac{1}{\cosh \left( \frac{V - v_3}{2v_4} \right)}$$  \hspace{1cm} (5)$$

where $v_1$, $v_2$, $v_3$, and $v_4$ are tuning parameters for steady state and time constant. Representative parameters for all equations (3)-(5) are given in Table 1 below.

Table 1: Parameter values of the Morris-Lecar oscillator model

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C$</td>
<td>$20 \mu F/cm^2$</td>
</tr>
<tr>
<td>$V_K$</td>
<td>$-84$ mV</td>
</tr>
<tr>
<td>$g_K$</td>
<td>$8$ mS/cm$^2$</td>
</tr>
<tr>
<td>$V_{Ca}$</td>
<td>$120$ mV</td>
</tr>
<tr>
<td>$g_{Ca}$</td>
<td>$4.4$ mS/cm$^2$</td>
</tr>
<tr>
<td>$V_L$</td>
<td>$-60$ mV</td>
</tr>
<tr>
<td>$g_L$</td>
<td>$2$ mS/cm$^2$</td>
</tr>
<tr>
<td>$v_1$</td>
<td>$-1.2$ mV</td>
</tr>
<tr>
<td>$v_2$</td>
<td>$18$ mV</td>
</tr>
<tr>
<td>$v_3$</td>
<td>$2$ mV</td>
</tr>
<tr>
<td>$v_4$</td>
<td>$30$ mV</td>
</tr>
<tr>
<td>$\phi$</td>
<td>$0.04$ /ms</td>
</tr>
</tbody>
</table>

Your tasks are the following:

1. (You may create a Matlab code or do this by hand): Calculate steady steady state points when
   (a) In the absence of the applied current.
   (b) The applied current is 60 pA.
   (c) The applied current is increased to 150 pA.

   [30 points]

2. (Matlab): Create a 2nd-order Runge-Kutta solver with $A = \frac{1}{5}$.

   [10 points]

3. (Matlab): Solve the Morris-Lecar problem at three conditions above (3 values of applied current) with the 2nd-order Runge-Kutta solver given the time span 0 to 200 ms, initial conditions $V_0 = 0.1$ and $w_0 = 0.7$, and step size $h = 2$. Plot the solutions.
4. (Matlab) Compare the solutions of $V$ and $w$ of the Morris-Lecar problem when applied current is 150 pA using the 2nd-order Runge-Kutta solver above and 4th-order Runge-Kutta solver. Create a table of comparison for $V$ and $w$ at times $t = 0, t = 20, t = 40, t = 60, t = 80, t = 100, t = 120, t = 140, t = 160, t = 180,$ and $t = 200$ ms.

[20 points]

Boundary Value Problems

Heat conduction at steady state along a rod with length $L = 1$ m is modeled by the following 2nd-order differential equations

$$\frac{d^2T}{dx^2} + 16\frac{dT}{dx} + 13T = 0$$

where temperatures at both ends of the rod are held constant at

$$T(0) = 3^\circ C \quad \text{and} \quad T(1) = 0.3^\circ C.$$ 

Solve the BVP using the shooting method in Matlab, by providing the following files

- One file for the equations, consisting a system of two 1st-order ODEs.
  [10 points]

- One file for running/solving the ODEs and plotting the results. Find the initial value of the two ODEs.
  [20 points]