Time-Varying Idiosyncratic Risk and Aggregate Consumption Dynamics

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Abstract

This paper presents an incomplete markets business cycle model in which the nature of idiosyncratic risk varies over time in accordance with recent empirical findings. In this model household heterogeneity and time-varying idiosyncratic risk have substantial effects on aggregate consumption dynamics. For example, the standard deviation of consumption growth is 47 percent larger than in a complete markets benchmark. Similarly, the decline in consumption in response to the labor market conditions that arose during the Great Recession is 50 percent larger than under complete markets.

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1 Introduction

Are idiosyncratic income risks important to the business cycle dynamics of aggregate quantities? Krusell and Smith (1998) provided a largely negative answer to this question. Their results show that the business cycle dynamics of their incomplete markets economy are generally close to those of a representative agent economy. I revisit this question with a focus on cyclical variation in uninsurable risks to an individual’s long-term earnings potential.

Recent empirical work using large panel datasets on individual earnings portray recessions as times when households face substantially larger downside risks to their earnings prospects. Moreover, these risks appear to have highly persistent effects on household earnings. Davis and von Wachter (2011) show that earnings losses from job-displacement are large, long-lasting, and roughly twice as large when the displacement occurs in a recession as opposed to an expansion. The differential impact of displacement in a recession is evident even twenty years after the event occurred. Similarly, Guvenen et al. (2013) show that the distribution of five-year earnings growth rates displays considerable pro-cyclical skewness meaning severe negative events are more likely in a recession. Similar cyclical risks have been estimated by Storesletten et al. (2004) using PSID data. Those authors find the standard deviation of idiosyncratic labor income shocks is much larger during recessions than it is during expansions.

The purpose of this paper is to investigate the implications of cyclical variation in idiosyncratic income risks for the business cycle dynamics of aggregate consumption. To do so, I develop a general equilibrium business cycle model with uninsurable idiosyncratic shocks to earnings. The idiosyncratic shock process is calibrated to match cyclical variation in the distribution of earning growth rates documented by Guvenen et al. (2013). In addition to time-varying idiosyncratic risk, the model also includes heterogeneity in time preference rates in the spirit of Krusell and Smith’s stochastic-β economy. This allows the model to generate a more realistic distribution of wealth and increases the number of households near the borrowing constraint.

The combination of time-varying idiosyncratic risk and borrowing constraints has a quan-
titatively substantial effect on the dynamics of aggregate consumption. In particular, the standard deviation of quarter-to-quarter aggregate consumption growth is 47 percent larger than it is in a complete markets version of the model. To illustrate the effects of household heterogeneity on aggregate consumption dynamics, I simulate the consumption response to estimates of the labor market shocks that occurred in the Great Recession of 2007-2009. These shocks include an increase in the unemployment rate, a negatively skewed distribution of innovations to household earnings and a decline in the job-finding rate. Relative to the NBER peak, aggregate consumption of non-durables and services was 3.6 percentage lower in 2009:II, which was the trough for this measure of consumption. The model presented here predicts a 2.6 percentage point drop in aggregate consumption by 2009:II. In contrast, the complete markets version of the model predicts a 1.4 percentage point drop by 2009:II.

In addition to the pronounced deterioration of labor market conditions, another defining feature of the Great Recession is the binding zero lower bound on short term interest rates. In modeling such episodes, a common way of bringing the economy to the zero lower bound is to introduce a wedge in the representative household’s Euler equation, typically in the form of a shock to the agent’s rate of time preference (e.g. Eggertsson and Woodford, 2003; Christiano et al., 2011; Eggertsson, 2011; Fernández-Villaverde et al., 2012). I ask whether precautionary savings and borrowing constraints can serve as a micro-foundation for this wedge. That a time-varying precautionary savings motive can manifest itself in a manner similar to a change in patience is well known (e.g. Braun and Nakajima, 2012). The purpose here is to quantify the extent of these effects. Specifically, I interpret the aggregate consumption path generated by the incomplete markets model through the lens of a representative agent model augmented with preference shocks. In order to fit the simulated data generated from incomplete markets model, the representative agent model requires a pronounced increase in patience starting in late 2008 that persists for several years. The quantitative magnitude of this increase in the annualized discount factor fluctuates around 3 percentage points. Christiano et al. (2011) find that changes in patience of this magnitude are enough to bring their model economy to the zero lower bound. Therefore, market incompleteness is capable of micro-founding an
aggregate Euler equation wedge of the magnitude assumed in the literature to cause the zero lower bound to bind.

The contribution of uninsurable idiosyncratic income risk to the dynamics of aggregate quantities has been studied previously and the most closely related contributions are Krusell and Smith (1998), Challe and Ragot (2013), and Ravn and Sterk (2013). This paper is primarily concerned with cyclical variation in highly-persistent earnings shocks as measured in the empirical papers mentioned above. By contrast, preceding work has incorporated cyclical variation in idiosyncratic risk through changes in the risk of unemployment, which is generally a short lived shock to earnings and therefore more easily smoothed through self-insurance. In this spirit, the work of Ravn and Sterk (2013) is notable as they allow for long-term unemployment spells and find substantial effects of precautionary savings motives on the dynamics of aggregate quantities. The calibration of the shock processes and cyclical variation in the risks is rather different in that approach.

The nature and source of cyclical changes in the earnings process are still somewhat poorly understood.\(^1\) This paper gives a particular interpretation to the facts on the distribution of earnings changes in expansions and recessions—these changes in income are uninsured and unforeseen risks—and then goes on to consider the implications for aggregate consumption dynamics. Other implications of this type of risk have also been studied. For example, Storesletten et al. (2007) investigate the asset pricing implications of this type of risk. The welfare cost of business cycles in the presence of these risks have been analyzed by Storesletten et al. (2001) and Krebs (2003, 2007). However, it does not appear that the consequences for the business cycle dynamics of aggregate quantities have been studied in the literature.

This paper is also related to the recent literature that investigates the role of uncertainty in business cycle fluctuations. In particular, Basu and Bundick (2012), Leduc and Liu (2012), and Fernández-Villaverde et al. (2013) emphasize the precautionary savings effect that follows an increase in uncertainty surrounding aggregate conditions such as preferences,

\(^1\)Davis and von Wachter (2011) show that structural models of the labor market have difficulty explaining the size and cyclicity of present-value earnings losses after job displacement. Huckfeldt (2014) presents a model that performs better but still struggles to explain the strong cyclicalality.
technology or taxes. In contrast to those studies, the focus here is on cyclical variation in microeconomic uncertainty faced by heterogeneous households. Other studies analyze the impact of cyclical microeconomic uncertainty faced by firms. This work is motivated by evidence of countercyclical dispersion in firm-level productivity, sales growth rates, and other measures of business conditions.\(^2\) Bloom (2009), Bloom et al. (2012) and Bachmann and Bayer (2013) study the interaction of this microeconomic uncertainty with non-convex adjustment costs for investment and hiring. Arellano et al. (2010) and Gilchrist et al. (2014) explore the interaction of firm-level risks and financial frictions. This paper contributes to this literature by studying the importance of variations in the microeconomic uncertainty surrounding household incomes for aggregate consumption.

The paper is organized as follows: Section 2 presents the model. Section 3 discusses the choice of parameters and computational methods. Section 4 presents the main results on the impact of household heterogeneity and time-varying earnings risks on the dynamics of aggregate consumption. Section 5 interprets market incompleteness as a foundation for aggregate time-preference shocks that might bring a representative agent model to the zero lower bound. Finally, the paper concludes with Section 6.

2 Model

I analyze a general equilibrium model with heterogeneous households and aggregate uncertainty. At the aggregate level, the model is similar to that of Krusell and Smith (1998). At the microeconomic level, I incorporate time-varying idiosyncratic risk with an income process similar to the one estimated by Guvenen et al. (2013).

2.1 Population, preferences and endowments

The economy is populated by a unit mass of households. Households survive from one period to the next with probability \(1 - \omega\) and each period a mass \(\omega \in (0, 1)\) of households is born.

\(^2\)See Bloom (2014) for a review of the evidence.
leaving the population size unchanged. At date 0, a household seeks to maximize preferences given by

$$E_0 \sum_{t=0}^{\infty} \beta^t (1 - \omega)^t \frac{C_t^{1-\gamma}}{1-\gamma},$$

where $C_t$ is the household’s consumption in period $t$. I allow for different rates of time preference across households in order to generate additional heterogeneity in wealth holdings.

Households can be either employed ($n = 1$) or unemployed ($n = 0$) and transition between these two states exogenously. Let $\lambda$ and $\zeta$ be the job-finding and -separation rates, respectively. Let $u \in [0, 1]$ be the unemployment rate.

If employed, a household exogenously supplies $e^y$ efficiency units of labor, where $y$ is the household’s individual efficiency. The cross-sectional dispersion in efficiency units could be due to differences in wage or due to differences in hours. For lack of a better term, I will refer to $y$ as “skill.” This skill evolves according to

$$y = \theta + \xi,$$

$$\theta' = \theta + \eta,$$

where $\xi$ is a transitory shock distributed $N(\mu_\xi, \sigma_\xi)$. I choose the constant parameters of the distribution for $\xi$ such that $E[e^\xi] = 1$. $\eta$ is a permanent shock to the individual’s skill. Assuming that this shock is permanent as opposed to persistent has the advantage that it allows one state variable to be eliminated from the household’s decision problem as described in Appendix C.\(^3\) This type of income process is known to fit longitudinal earnings data well as shown by MaCurdy (1982) and Abowd and Card (1989).

The permanent shock, $\eta$, is drawn from a time-varying distribution the tails of which vary over the business cycle in such a way to generate pro-cyclical skewness as documented in the data by Guvenen et al. (2013). Like Guvenen et al., I assume $\eta$ is drawn from a

\(^3\)See also Carroll et al. (2013).
mixture of normals. Specifically

\[ \eta \sim \begin{cases} 
N(\mu_{1,t}, \sigma_{\eta,1}) & \text{with prob. } p_1 \\
N(\mu_{2,t}, \sigma_{\eta,2}) & \text{with prob. } p_2 \\
N(\mu_{3,t}, \sigma_{\eta,3}) & \text{with prob. } p_3,
\end{cases} \]

where \( \sum_{j=1}^{3} p_j = 1 \). The time-varying parameters \( \mu_{1,t}, \mu_{2,t}, \) and \( \mu_{3,t} \) will, respectively, control the center, right tail, and left tail of the distribution. I assume that these distributional parameters are driven by a single stochastic process, \( x_t \), according to

\begin{align*}
\mu_{1,t} &= \bar{\mu}_t \\
\mu_{2,t} &= \bar{\mu}_t + \mu_2 - x_t \\
\mu_{3,t} &= \bar{\mu}_t + \mu_3 - x_t,
\end{align*}

An increase in \( x_t \) moves the tails of the distribution to the left relative to the center of the distribution and will generate negative skewness in the distribution. \( \bar{\mu}_t \) is a normalization such that \( \mathbb{E}[e^{\eta}] = 1 \) in all periods. This normalization in turn implies that \( \mathbb{E}[e^{\theta}] = 1 \) given suitable initial conditions.\(^4\) The details of this normalization appear in Appendix B.

As the three idiosyncratic labor income shocks—\( \theta, \xi \) and \( n \)—are independent, using a law of large numbers the aggregate labor input is

\[ \bar{L} \equiv \mathbb{E} [e^{\theta + \xi n}] = \mathbb{E} [e^{\theta}] \mathbb{E} [e^{\xi}] (1 - u) = 1 - u. \]

It would be natural to assume that there is a correlation between shocks to skill and shocks to employment. I have experimented with including such a correlation and found that it has little impact on the results.

\(^4\)To verify this observe that

\[ \mathbb{E}[e^{\theta}] = (1 - \omega)\mathbb{E}[e^{\theta + \eta}] + \omega = (1 - \omega)\mathbb{E}[e^{\theta}]\mathbb{E}[e^{\eta}] + \omega = (1 - \omega)\mathbb{E}[e^{\theta}] + \omega, \]

which implies that \( \mathbb{E}[e^{\theta}] \) converges to one.
It is important that the model includes mortality risk, which allows for a finite cross-sectional variance of skills despite the fact that innovations to skills are permanent. When a household dies, it is replaced by a newborn household with no assets and skill, $e^\theta$, normalized to one. The unemployment rate among newborn households is the same as prevails in the surviving population at that date. A household’s rate of time preference is fixed throughout its life and drawn initially from a stable distribution.

2.2 Technology, markets, and government

A composite good is produced out of capital and labor according to

$$\bar{Y} = e^z \bar{K}^\alpha \bar{L}^{1-\alpha}$$

where $z$ is an exogenous total factor productivity (TFP) and aggregate quantities are denoted with a bar. Capital depreciates at rate $\delta$ and evolves according to

$$\bar{C} + \bar{K}' = \bar{Y} + (1 - \delta)\bar{K}.$$  

The factors of production are rented from the households each period at prices that satisfy the representative firm’s static profit maximization problem

$$W = (1 - \alpha)e^z \bar{K}^\alpha \bar{L}^{-\alpha}$$

$$\bar{R} = \alpha e^z \bar{K}^{\alpha-1}\bar{L}^{1-\alpha} + 1 - \delta.$$  

Here $\bar{R}$ is the return on capital and $W$ is the wage paid per efficiency unit. Households save in the form of annuities and the return to surviving households is $R \equiv \bar{R}/(1 - \omega)$. I assume that savings must be non-negative due to borrowing constraints. Given the income process, the zero borrowing limit is the natural borrowing limit.

The data reported by Guvenen et al. (2013) refer to pre-tax earnings. As taxes and transfers provide insurance against idiosyncratic risks it is important to incorporate this
insurance into the model. Let the net tax payment of an employed individual with earnings \( W e^y \) be \( W e^y - (1 - \tau)W e^{(1-b^y)y} \). The parameters \( \tau \) and \( b^y \) control the level and progressivity of the tax, respectively. For incomes less than \( (1 - \tau)^{1/b^y} \) the average tax rate is negative and the household receives a transfer from the government. Heathcote et al. (2014) discuss the properties of this type of tax system in detail.

Unemployed households receive taxable unemployment insurance payments with a replacement rate \( b^u \). The post-government income of a household with employment status \( n \in \{0, 1\} \) and skill \( e^y \) is therefore

\[
(1 - \tau)W e^{(1-b^y)y}[n + b^u(1 - n)].
\]  

I assume the level of the tax system, \( \tau \), is adjusted to balance the budget of the tax and transfer system period by period, which requires

\[
1 - \tau = \frac{1 - u}{Q(1 - u + b^u u)} \quad (8)
\]

where \( Q \equiv \mathbb{E}[e^{y(1-b^y)}] \) reflects the fact that a progressive income tax raises more revenue when incomes are more dispersed. As explained in Appendix A, \( Q \) evolves according to

\[
Q' = (1 - \omega)Q\tilde{Q}^\nu + \omega\tilde{Q}^\xi
\]

where \( \tilde{Q}_\xi \equiv \mathbb{E}[e^{(1-b^y)\xi}] \) and \( \tilde{Q}^\nu \equiv \mathbb{E}[e^{(1-b^y)\nu}] \).

### 2.3 Aggregate shock processes

I assume the following processes for aggregate shocks. TFP evolves according to

\[
z' = \rho_z z + \epsilon'_z.
\]  

For the labor market, I assume that aggregate shocks occur at the start of a period and labor market outcomes in period \( t \) reflect the shocks realized at date \( t \). I assume that the
unemployment rate and job-finding rate follow AR(1) processes with correlated innovations. Specifically,

\[
\hat{u}' = (1 - \rho_u)\hat{u}^* + \rho_u \hat{u} + \epsilon_u \tag{11}
\]

\[
\hat{\lambda}' = (1 - \rho_\lambda)\hat{\lambda}^* + \rho_\lambda \hat{\lambda} + \epsilon_\lambda, \tag{12}
\]

where \(\hat{u}\) is the inverse-logistic transformation\(^5\) of the unemployment rate and \(\hat{\lambda}\) is similarly defined. \(\hat{u}^*\) and \(\hat{\lambda}^*\) are constant parameters that determine the mean unemployment and job-finding rates, respectively. The job-separation rate, \(\zeta\), is determined implicitly by the law of motion

\[
u' = (1 - \nu')u + \zeta'(1 - u). \tag{13}\]

The process for skill risk, \(x\), follows

\[
x' = \rho_x x + \epsilon_x, \tag{14}\]

where the innovations, \(\epsilon_x\), are correlated with \(\epsilon_u\) and \(\epsilon_\lambda\).

### 2.4 The household’s decision problem

The individual state variables of the household’s decision problem are its cash on hand, call it \(A\), its permanent skill, \(\theta\), and its employment status, \(n\). In addition households differ in their rates of time preference although these are not state variables as they are fixed within a household’s lifetime. The aggregate states are \(S \equiv \{z, \lambda, u_{-1}, x, \Gamma\}\), where \(\Gamma\) is the distribution of households over the state space from which one can calculate aggregate capital, \(\bar{K}\), \(Q\), and the unemployment rate, \(u\). The lagged unemployment rate, \(u_{-1}\) is needed in order to calculate the job-separation probability. Appendix C describes how the model can be normalized to eliminate some of these state variables. The household’s decision variable

\(^5\)That is, \(u\) and \(\hat{u}\) are related according to \(u = 1/(1 + e^{-\hat{u}})\).
is end-of-period savings, $K'$.

The household’s decision problem is then

$$V(A, \theta, n, S) = \max_{K' \geq 0} \left\{ \frac{(A - K')^{1-\gamma}}{1-\gamma} + \beta(1 - \omega)\mathbb{E}[V(A', \theta', n', S')] \right\}$$

subject to

$$A' = R(S')K' + (1 - \tau(S'))W(S')e^{(1-b')(\theta + \eta' + \xi')} [n' + b^u(1 - n')] .$$

The prices in the household’s problem depend on the aggregate state $S$ through (5) and (6). The law of motion for the aggregate state is given by (10), (11), (12), (14), $u'_{-1} = u$, and a law of motion for the distribution of idiosyncratic states, $\Gamma' = H_{\Gamma}(S, u', \lambda')$.

## 2.5 Equilibrium

Let $F(A, \theta, n, S)$ be the optimal decision rule for $K'$ in the household’s problem. Aggregate savings are

$$K' = \sum_n \int_K \int_{\theta} \int_{\xi} F(RK + (1 - \tau)W e^{(1-b')(\theta + \xi)} [n + b^u(1 - n)], \theta, n, S) \Phi(d\xi) \Gamma(dK, d\theta, n),$$

(15)

where $\Phi$ is the distribution of $\xi$. Given a set of exogenous stochastic processes for $z$, $u$, $\lambda$, and $x$, a recursive competitive equilibrium consists of the law of motion for the distribution, $H_{\Gamma}$, household value function, $V$, and policy rule, $F$, and pricing functions $W$ and $R$. In an equilibrium, $V$ and $F$ are optimal for the household’s problem, $R = \tilde{R}/(1 - \omega)$ and $W$ satisfy (5)-(6), and $H_{\Gamma}$ is induced by $F$ and the idiosyncratic income process.
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>Capital share</td>
<td>0.36</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Depreciation rate</td>
<td>0.02</td>
</tr>
<tr>
<td>$\rho_z$</td>
<td>Persistence of TFP</td>
<td>0.96</td>
</tr>
<tr>
<td>$\sigma_z$</td>
<td>St. dev. of TFP innovation</td>
<td>0.0081</td>
</tr>
<tr>
<td>$\omega$</td>
<td>Mortality rate</td>
<td>0.005</td>
</tr>
<tr>
<td>$b^u$</td>
<td>Unemployment insurance replacement rate</td>
<td>0.30</td>
</tr>
<tr>
<td>$b^y$</td>
<td>Tax-and-transfer progressivity</td>
<td>0.151</td>
</tr>
<tr>
<td>$\beta_{\text{low}}$</td>
<td>Discount factor</td>
<td>0.920</td>
</tr>
<tr>
<td>$\beta_{\text{high}}$</td>
<td>Discount factor</td>
<td>0.982</td>
</tr>
<tr>
<td>$\mu_2$</td>
<td>Mean of right tail of $\eta$ distribution</td>
<td>0.355</td>
</tr>
<tr>
<td>$\mu_3$</td>
<td>Mean of left tail of $\eta$ distribution</td>
<td>-0.2989</td>
</tr>
<tr>
<td>$\sigma_{1,\eta}$</td>
<td>St. dev. of center of $\eta$ distribution</td>
<td>0.0143</td>
</tr>
<tr>
<td>$\sigma_{2,\eta}$</td>
<td>St. dev. of right tail of $\eta$ distribution</td>
<td>0.1041</td>
</tr>
<tr>
<td>$\sigma_{3,\eta}$</td>
<td>St. dev. of left tail of $\eta$ distribution</td>
<td>0.1041</td>
</tr>
<tr>
<td>$\sigma_\xi$</td>
<td>St. dev. of transitory income shock</td>
<td>0.1580</td>
</tr>
<tr>
<td>$p_1$</td>
<td>Weight of center of $\eta$ distribution</td>
<td>0.8948</td>
</tr>
<tr>
<td>$p_2$</td>
<td>Weight of right tail of $\eta$ distribution</td>
<td>0.0526</td>
</tr>
<tr>
<td>$p_3$</td>
<td>Weight of left tail of $\eta$ distribution</td>
<td>0.0526</td>
</tr>
</tbody>
</table>

Table 1: Calibrated parameter values.
3 Parameters and computation

I begin by describing the calibration of the income process before turning to the other parameters of the model and finally the computational methods.

3.1 The idiosyncratic income process

Calibrating the model requires an empirical counterpart to the variable $x_t$ in the model, which changes the distribution of idiosyncratic risk and I construct this using a simulated method of moments procedure. The empirical moments describe the year-by-year distribution of one-year, three-year, and five-year earnings changes reported by Guvenen et al. (2013). The assumption underlying my approach is that developments in the labor market drive both $x_t$ and observable indicators of labor market conditions. I use four such indicators: the ratio of short-term unemployed (fewer than 15 weeks) to the labor force, the same ratio for long-term unemployed (15 or more weeks), an index of average weekly hours, and the labor force participation rate. Note that the employment-population ratio can be expressed as a function of these variables. I then posit that $x_t$ is a linear combination of these four series. In addition to the factor loadings for $x$, I simultaneously determine values for $p_2$, $p_3$, $\mu_2$, $\mu_3$, $\sigma_{1,\eta}$, $\sigma_{2,\eta}$, $\sigma_{3,\eta}$, and $\sigma_\xi$ while imposing the restrictions $p_3 = p_2$ and $\sigma_{2,\eta} = \sigma_{3,\eta}$.

For each candidate parameter vector, I simulate the income process for a panel of households including employment and mortality shocks and form an objective function that penalizes the distance between the model-implied moments and the empirical moments. The moments I seek to match are the year-by-year values for the median, 10th percentile and 90th percentile of the one-year, three-year and five-year earnings growth distributions. The Guvenen et al. data range from 1978 to 2011 and in total there are 279 moments.

To simulate the model, I need estimates of $\lambda_t$ and $\zeta_t$. I estimate these from the relation-
where $u_t$ is the unemployment rate and $u^s_t$ is the short-term unemployment rate measured as those with durations less than 15 weeks. I simulate quarterly data and then aggregate to annual observations to conform to the Guvenen et al. data. Appendix B contains further discussion of the implementation of this method.

Figure 1 shows the model’s fit to the earnings growth distribution at one-year, three-year and five-year horizons. The model does a good job of matching the moments of the three-year and five-year earnings changes. While the model fails to generate the volatility of the 10th and 90th percentiles for one-year changes, this is not too worrisome as the three-year and five-year earnings changes are a better reflection of long-term earnings risks that are of particular interest here.

The left panel of Figure 2 shows the PDF of $\eta$ for $x = 0$. There is a large mass near zero and dispersed tails. The right panel of Figure 2 shows the effect of an increase in $x$ to 0.2 on the distribution of $\eta$ with the vertical axis of the figure scaled to emphasize the tails of the distribution. The negative skewness caused by $x = 0.2$ is evident as the left tail now shifts away from the central mass and the right tail compresses towards it.

The left panel of Figure 3 shows the time series for $x_t$ that is generated by this procedure. One can see that there are sharp spikes in this measure of idiosyncratic risk during recessions. The correlation of this series with short-term unemployment is 0.8. As short-term unemployment is high when workers are flowing into unemployment during recessions one interpretation is that labor market events that lead to flows into unemployment are also associated with negatively skewed innovations in permanent skill. This interpretation is consistent with the findings of Davis and von Wachter (2011) who present evidence that job layoffs are associated with large and long-lasting reductions in earnings and that long-term earnings losses are roughly twice as large for layoffs that occur in recessions.
Figure 1: Simulated (dark line) and empirical (light line) moments of the earnings process.
Figure 2: PDF for distribution of $\eta$ for $x = 0$ and $x = 0.2$.

Figure 3: Empirical measure of $x_t$ and Kelley’s skewness of five-year earnings changes for model (solid) and data (dashed).

The right panel of Figure 3 shows a measure of skewness in the five-year earnings changes for the model and the data. Kelley’s skewness is Guvenen et al.’s preferred measure of skewness because it is less sensitive to extreme observations. It is calculated from the 10th, 50th and 90th percentiles of the distribution as $((P_{90} - P_{50}) - (P_{50} - P_{10}))/((P_{90} - P_{10})$. The model slightly understates the volatility in this measure of risk.

To parameterize the aggregate shock processes, I estimate AR(1) processes for the three series $\hat{u}_t$, $\lambda_t$, and $x_t$. Given these estimates, I calculate the covariance matrix of the residuals
and perform a Cholesky decomposition of the covariance matrix yielding the following system

\[
[\hat{u}' - 0.0638, \lambda' - 0.7679, x']^T = D [\hat{u} - 0.0638, \lambda - 0.7679, x]^T + \epsilon',
\]

where \(D\) is a diagonal matrix with diagonal elements \([0.9848, 0.9457, 0.9169]\) and the decomposed covariance matrix of \(\epsilon\) is

\[
\begin{pmatrix}
0.0033 & 0 & 0 \\
-0.0626 & 0.0563 & 0 \\
0.0153 & 0.0121 & 0.0315
\end{pmatrix}.
\]

### 3.2 Other parameters

I set the remaining parameters of the model to match well known targets. The coefficient of relative risk aversion is set to 2, the depreciation rate is set to 2 percent per quarter. I set the persistence of the productivity process to 0.96 in line with typical estimates for the US. The labor share is set to 64 percent and the mortality risk is set to 0.5 percent per quarter for an expected working lifetime of 50 years.

I set the unemployment insurance replacement rate, \(b_u\), to 0.3, which is in line with replacement rates for the United States reported by Martin (1996). The skill insurance parameter \(b_y\) is set to 0.151, which is the progressivity of the tax-and-transfer system estimated by Heathcote et al. (2014) to fit the relationship between pre- and post-government income in PSID data.\(^6\)

I assume that there are two values of \(\beta_i\) in the population with 40 percent of the population having the lower value and 60 percent having the higher value. I choose these values,\(^6\)
<table>
<thead>
<tr>
<th>Share of wealth by quintile</th>
<th>Gini</th>
<th>Average MPC</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st</td>
<td>2nd</td>
<td>3rd</td>
</tr>
<tr>
<td>Baseline</td>
<td>0.001</td>
<td>0.008</td>
</tr>
<tr>
<td>Common-β</td>
<td>0.031</td>
<td>0.074</td>
</tr>
<tr>
<td>Data</td>
<td>-0.002</td>
<td>0.011</td>
</tr>
</tbody>
</table>

Table 2: Distribution of wealth and average MPC. Data are from the 2007 Survey of Consumer Finances as reported by Diaz-Gimenez et al. (2011).

and the volatility of the productivity process to match the following moments in an internal calibration: a capital-output ratio of 3.32, the wealth share of the bottom 40 percent by wealth equal to 0.9 percent of total wealth (see Diaz-Gimenez et al., 2011), and the standard deviation of log output growth equal to 0.0084. The resulting parameter values appear in Table 1 and the model generated distribution of wealth appears in Table 2.

The calibration of the discount rates is in one sense extreme. By targeting the wealth holdings of the bottom 40 percent while not allowing them to take debt, I am perhaps exaggerating how close they are to the borrowing constraint. On the other hand, there is a sense in which this calibration is not extreme enough. The average marginal propensity to consume in this model is 10 percent in the model’s risky steady state. Empirical studies typically find values of the MPC in the range of 20 percent to 50 percent with many estimates near 20 percent. If I sought to match the response of consumption to income changes then I would need to have more households with consumption closely tied to income.

### 3.3 Computation

The model presents two computational challenges. First, the aggregate state of the model includes the endogenous distribution of households over individual states. I use the Krusell-Smith algorithm and replace this distribution with the first moment for capital holdings, Coeurdacier et al. (2013) define the risky steady state as the point to which the economy will converge if the realization of aggregate shocks is zero for all periods. This concept differs from the deterministic steady state in that the agents believe that aggregate shocks can occur and this contributes to their precautionary savings motive.

Examples include Hall and Mishkin (1982); McCarthy (1995); Lusardi (1996); Parker (1999).
\( \bar{K} \), the unemployment rate, \( u \), and the measure of income inequality, \( Q \). The aggregate state is then \( S_t = \{ z, u, \bar{K}, \lambda, u_{-1}, x, Q \} \), which is seven continuous variables. The second computational challenge is the curse of dimensionality as the model includes seven aggregate states, three individual states and four aggregate shocks.\(^9\) To compute solutions to the household’s problem efficiently, I make use of the algorithm introduced by Judd et al. (2012) to construct a grid on the part of the aggregate state space that the system actually visits. This approach reduces the computational cost of having many state variables while still allowing for accurate solutions by avoiding computing the solution for combinations of states that are very unlikely to arise in practice. For individual cash on hand, I use an endogenous grid point method and place 100 grid points on \( K' \). Appendix D provides further discussion of the methods and presents several accuracy checks.

4 Idiosyncratic risk and aggregate consumption dynamics

I now assess the extent to which household heterogeneity and uninsurable idiosyncratic risk alters the dynamics of aggregate consumption. To do so, I make use of three benchmarks: (i) with a common rate of time preference and therefore less heterogeneity in wealth, (ii) with a common rate of time preference and no shocks to the skewness of the income process,\(^{10}\) and (iii) a complete markets version of the model. In the complete markets model, households have a common discount rate and all shocks are insurable including mortality risk, which leads to the standard Euler equation for aggregate consumption

\[
\bar{C}^{-\gamma} = \beta \mathbb{E}_t \left[ \bar{C}'^{-\gamma} \bar{R}' \right]
\]

\(^{9}\)There are three individual states as opposed to four because the household’s problem is homogeneous in \( \exp \{(1 - b^\theta)\theta \} \) so it is sufficient to normalize cash on hand by this value and eliminate one state.

\(^{10}\)For this case, I solve the same model as in the baseline case, but analyze the dynamics of the economy for realizations of the aggregate shocks in which \( x_t = 0 \) for all \( t \).
as shown in Appendix E. For both the model without time-preference heterogeneity and the complete markets model I recalibrate the discount rate to match the capital-output ratio from the baseline model.

Table 4 displays standard deviations and correlations of output and consumption both in log-levels and in growth rates. Notice that the standard deviation of consumption growth is 47 percent larger in the baseline model than in the complete markets version of the model. Comparing the model with a common rate of time preference and no shocks to the skewness of the income process (row iii) to the complete markets model (row iv) the difference in standard deviations is only 4 percent. This similarity arises despite the fact that this version of the model still contains considerable heterogeneity in wealth and income. This result resembles the baseline findings in Krusell and Smith (1998).

There are two main sources of difference between the baseline model and the complete markets benchmark. One is the time-varying precautionary savings motive and the second is a high MPC in the baseline model, which derives from the calibration of the time-preference rates. To assess the contribution of time-varying idiosyncratic risk in the absence of the high MPCs generated by heterogeneity in time preference rates, one can compare rows (ii) and (iii) of Table 4. Here one can see that time-varying risk raises the standard deviation of consumption growth by 20 percent.

The standard deviation of the log-level of consumption is much less different across versions of the model than is the standard deviation of growth rates differing by only six percent between the baseline model and the complete markets model. Much of the variation in the level of consumption is driven by low-frequency changes in productivity and capital. Around this path, changes in the precautionary savings motive introduce additional dynamics in consumption. As the fluctuations in the precautionary savings motive are short-lived, see Figure 3, the induced dynamics in consumption are short-lived and therefore do not much affect the level of consumption.

---

11Simulated consumption growth is especially sensitive to sampling variation. For a fixed set of aggregate shocks, I continue increasing the number of households in the simulation until the standard deviation of consumption growth stabilizes. For the baseline model this required simulating a panel of 7.2 million households.
<table>
<thead>
<tr>
<th></th>
<th>$\sigma_Y$</th>
<th>$\sigma_C$</th>
<th>$\rho_{Y,C}$</th>
<th>$\sigma_{\Delta Y}$</th>
<th>$\sigma_{\Delta C}$</th>
<th>$\rho_{\Delta Y, \Delta C}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(i) Baseline</td>
<td>0.0448</td>
<td>0.0352</td>
<td>0.969</td>
<td>0.00843</td>
<td>0.00437</td>
<td>0.934</td>
</tr>
<tr>
<td>(ii) Common-(\beta)</td>
<td>0.0474</td>
<td>0.0356</td>
<td>0.936</td>
<td>0.00843</td>
<td>0.00343</td>
<td>0.781</td>
</tr>
<tr>
<td>(iii) Common-(\beta), constant risk</td>
<td>0.0475</td>
<td>0.0354</td>
<td>0.944</td>
<td>0.00843</td>
<td>0.00285</td>
<td>0.965</td>
</tr>
<tr>
<td>(iv) Complete markets</td>
<td>0.0461</td>
<td>0.0331</td>
<td>0.948</td>
<td>0.00843</td>
<td>0.00297</td>
<td>0.986</td>
</tr>
</tbody>
</table>

Table 3: Standard deviations ($\sigma$) and correlations ($\rho$) of aggregate output ($Y$) and consumption ($C$) log-levels and growth rates (denoted with $\Delta$).

Turning to the correlation between the levels of aggregate consumption and aggregate income, one can see that it is highest in the baseline model, which reflects the strong co-movement of consumption and income for households near the borrowing constraint. The baseline model yields an average MPC of 10 percent, which is considerably larger than that for the model with a single rate of time preference where it is 2 percent. The precautionary-saving motive introduced by time-varying idiosyncratic risk is only weakly correlated with aggregate income and so the dynamics it induces serve to reduce the correlation of aggregate consumption and income. This force is seen most easily by comparing growth rates in rows (ii) and (iii) of the table.

In this model the only way that household choices affect aggregate income dynamics is through the aggregate capital stock and the dynamics of aggregate output are little affected by household heterogeneity. This is especially true of output growth, which is dominated by exogenous shocks.

Do the results in Table 4 reflect important differences across model specifications? To illustrate the meaning of these differences I consider the consumption response to the labor market shocks that occurred during the Great Recession of 2007 - 2009. I assume that the economy is in its risky steady state in 2007:I. Beginning from this starting point, I simulate the economy using the shocks taken from the data for the unemployment rate, $u$, the skewness of the income shock process, $x$, and the job-finding rate, $\lambda$. The construction of the series for $x$ and $\lambda$ is described in Section 3. Given the assumption about the initial condition in 2007:I, I then use equations (11), (12), and (14) to solve for sequences of $\epsilon_{u,t}$,
The top panel of Figure 4 plots the path for consumption starting in 2007:II and normalized to one in 2007:IV, which was the peak of the expansion as defined by the NBER. In addition to the four versions of the model considered above, the figure also plots the data on aggregate consumption of services and non-durable goods detrended with the HP filter. Four aspects of the figure stand out. First, the baseline model generates a drop in consumption that is nearly as large as observed in the data despite the fact that the only shocks considered are to the labor market. Second, the version of the model with a common rate of time preference and constant risk generates dynamics for consumption that are very close to the complete markets model. Third, time-varying risk contributes to the sharp drop in 2008:IV, but then the effect of time-varying risk dies out relatively quickly (compare common- to common- and constant risk). Finally, the protracted decline in aggregate consumption mirrors the path of the unemployment rate in the baseline model.

The close relationship between consumption and labor income in the baseline model reflects the behavior of low-wealth households who do not effectively smooth their consumption.

5 Market incompleteness as a foundation for preference shocks

Recent events have stimulated a great deal of interest in understanding the behavior of economies at the zero lower bound on nominal interest rates. In modeling these episodes, a common way of bringing the economy to the zero lower bound is to subject it to a shock to the time preference of agents in the model (Eggertsson and Woodford, 2003; Christiano et al., 2011; Eggertsson, 2011; Fernández-Villaverde et al., 2012). This type of shock is useful for this purpose because it generates a wedge in the representative household’s Euler equation that reduces aggregate consumption and interest rates simultaneously. It is clear that in this modeling approach time-preference shocks are standing in for other forces in the economy. In this section, I evaluate the extent to which the precautionary savings behavior and binding
Figure 4: Dynamics of aggregate consumption implied by labor market shocks in the Great Recession. Data refer to per capita consumption of non-durables and services deflated with the GDP deflator and smoothed with the HP filter with smoothing parameter 1600.
borrowing constraints present in the incomplete markets model provide a foundation for these time-preference shocks. To do so, I interpret the consumption profile from the baseline incomplete markets model in Figure 4 through the lens of a representative agent model augmented with time-preference shocks.

The representative household’s utility function is now

\[ E_0 \sum_{t=0}^{\infty} \left( \prod_{\tau=0}^{t} \beta_{\tau} \right) \frac{C_{t}^{1-\gamma}}{1-\gamma}, \]

where \( \beta_t \) is a preference shock that evolves according to

\[ \log \beta_t - \log(\bar{\beta}) = \rho_\beta (\log \beta_{t-1} - \log(\bar{\beta})) + \epsilon_{\beta,t}. \]

The remaining features of the model are unchanged. I assume that \( \rho_\beta = 0.8 \) and \( \bar{\beta} = 0.9880 \), which is the same value as for complete markets economy from Section 4. In solving for the household’s decision rule I assume that the variance of \( \epsilon_{\beta,t} \) is zero so there is no precautionary savings motive generated by these preference shocks.\(^{12}\)

I use this model to interpret the behavior of aggregate consumption generated by the baseline model in response to the labor market shocks during the Great Recession as shown in Figure 4. Specifically, I feed the same labor market shocks as used in Section 4 into the representative agent model and calculate the level of preference shock that would lead the representative household to choose the consumption path that the incomplete markets economy generates.

Figure 5 shows the sequence of \( \beta_t \) that is needed in the complete markets model to explain the consumption series generated by the incomplete markets model. Precautionary savings and binding borrowing constraints generate dynamics for aggregate consumption that are as if the representative agent became much more patient in late 2008 and remained more

\[^{12}\]I use the same algorithm to solve this model as described above, which involves simulating the model to compute a grid for the state space. In this simulation I set the standard deviation of \( \epsilon_{\beta,t} \) to 0.006 so that there is dispersion in the grid points in this dimension that covers the range of values needed in the analysis below.
Figure 5: Representative agent’s rate of time preference relative to steady state needed to rationalize the baseline incomplete markets consumption path in Figure 4.

patient for several years there after.

To interpret the quantitative significance of this level of time preference shock, consider the analysis of Christiano et al. (2011) who find a 2 percentage point shock to preferences that persists from quarter to quarter with 80 percent probability brings their model economy to the zero lower bound. The path shown in Figure 5 shows that the implied discount factor shock exceeds this level for several years starting in 2008:IV. While it is beyond the scope of this paper to analyze the economy at the zero lower bound, it appears that household heterogeneity with time-varying idiosyncratic risk and binding borrowing constraints can generate a wedge in the aggregate Euler equation of a sufficient magnitude to provide a micro-foundation for the size of shocks used in the literature.

6 Conclusion

This paper began by asking whether idiosyncratic risks are relevant for the dynamics of aggregate quantities. The results show that a combination of time-varying risks to long-term
earnings potential and the high marginal propensity to consume of low-wealth households can generate substantial movements in aggregate consumption. The model predicts that the standard deviation of aggregate consumption growth is 47 percent larger as result of these forces as compared to a complete markets benchmark.

The difference between the aggregate consumption dynamics generated by the incomplete markets and complete markets models is less pronounced if one compares levels as opposed to growth rates. For example, the standard deviation of the level of consumption is only six percent larger than in the complete markets model. The level of consumption reflects low-frequency developments more strongly than growth rates do. As the extent of idiosyncratic risk appears to spike in recessions and quickly recede to more normal levels, its effects on the level of consumption are more muted than its effects on growth rates. The focus on growth rates is one reason that my conclusions differ from those of Krusell and Smith (1998). Whether levels or growth rates are the appropriate object of study depends on the use one has for the model with high-frequency considerations favoring growth rates.

This paper has focussed on the dynamics of aggregate consumption. At the aggregate level, the model is a version of the flexible-price real business cycle model with exogenous labor supply and as a result an increase in household savings necessarily leads to an increase in investment and an increase in output in future periods. A richer model could allow the forces studied here to generate business cycle fluctuations in consumption, output, hours and investment. In the context of a representative agent model, Basu and Bundick (2012) show that a precautionary savings motive can lead to business cycle co-movements in the presence of nominal rigidities. A recent strand of literature incorporates nominal rigidities into incomplete markets models with with heterogeneous households (Guerrieri and Lorenzoni, 2011; Oh and Reis, 2012; McKay and Reis, 2013; Ravn and Sterk, 2013; Gornemann et al., 2012). The current paper contributes to this literature by identifying a set of forces by which household heterogeneity has important consequences for aggregate consumption. Future work might enrich the model presented here to draw out the implications of these consumption dynamics for other components of the business cycle.
References


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Appendix

A Dynamics of $Q$

To calculate the dynamics of the tax adjustment, $Q$, in equation (8) define

\[
\tilde{Q}^\theta = \mathbb{E}\left[e^{\theta(1-b^y)}\right]
\]
\[
\tilde{Q}^\eta = \mathbb{E}\left[e^{\eta(1-b^y)}\right]
\]
\[
\tilde{Q}^\xi = \mathbb{E}\left[e^{\xi(1-b^y)}\right],
\]

where expectations are taken across agents. By the independence of the shocks one can write

\[
Q = \tilde{Q}^\theta \tilde{Q}^\xi.
\]

$\tilde{Q}^\theta$ evolves according to

\[
\tilde{Q}^{\theta'} = (1 - \omega)\mathbb{E}\left[e^{(\theta + \eta')(1-b^y)}\right] + \omega
\]
\[
\tilde{Q}^{\theta'} = (1 - \omega)\tilde{Q}^{\theta} \tilde{Q}^{\eta'} + \omega.
\]

And as $\tilde{Q}^\xi$ is constant one can then write

\[
\tilde{Q}^{\theta'} \tilde{Q}^\xi = (1 - \omega)\tilde{Q}^{\theta} \tilde{Q}^{\eta'} \tilde{Q}^\xi + \omega \tilde{Q}^\xi
\]
\[
Q' = (1 - \omega)Q \tilde{Q}^{\eta'} + \omega \tilde{Q}^\xi.
\]

B Calibrating the idiosyncratic income process

This appendix provides additional information on the simulated method of moments procedure used to select the parameters of the idiosyncratic income process, which is a variant of the procedure used by Guvenen et al. (2013).
Step 1. Calculate $\lambda_t$ and $\zeta_t$ implied by the data. To do so, use the data on short-term unemployment described in Section 3 and solve for $\lambda_t$ and $\zeta_t$ from equations (16) and (17).

Step 2. Construct the four labor market indicators. I use four such indicators: the ratio of short-term unemployed (fewer than 15 weeks) to the labor force, the same ratio for long-term unemployed (15 or more weeks), an index of average weekly hours, and the labor force participation rate.\textsuperscript{13} Note that the employment-population ratio can be expressed as a function of these variables. I transform these four series to have mean zero and unit standard deviation and then express the resulting series in terms of their principal components. Orthogonalizing the series into principal components should not affect the results in theory, but it is helpful for the numerical analysis. These quarterly data cover 1977:I to 2011:IV. Store these in a matrix $X$.

Step 3. Guess a vector of parameters

$$\Theta \equiv [\phi_1, \cdots, \phi_4, \sigma_\xi, \mu_2, \mu_3, \sigma_{\eta,1}, \sigma_{\eta,2}, \sigma_{\eta,3}, p_2],$$

and $\phi_j$ is a loading on the $j$th labor market indicator. Also guess a sequence $\{m_t\}_{t=1978}^{2011}$. $m_t$ is the quarterly growth rate of average income in year $t$, which shifts the entire distribution from which $\eta$ is drawn. While I simulate quarterly data, I assume the mean growth rate is constant in each year as the observed data are at an annual frequency.

Step 4. Calculate $\mu_{1,t}$, $\mu_{2,t}$ and $\mu_{3,t}$ from Equations (1) - (3) with $x = X\phi$. The normalization $\bar{\mu}$ is chosen to satisfy $E[e^\eta] = 1$ and this requires

$$\bar{\mu} = -\log \left( p_1 \exp(\sigma_{\eta,1}^2/2) + p_2 \exp(\mu_2 - x + \sigma_{\eta,2}^2/2) + p_3 \exp(\mu_3 - x + \sigma_{\eta,3}^2/2) \right). \quad (A1)$$

\textsuperscript{13}These data series constructed from the series with the following codes in the Federal Reserve Bank of St. Louis FRED database: CLF16OV, UNEMPLOY, UEMP15OV, PRS85006023, and CIVPART.
Step 5. Simulate employment, skill, and mortality shocks for a panel of households. The employment transition probabilities are the values for $\lambda_t$ and $\zeta_t$ computed in step 1. I simulate 10,000 individuals from 1977 through 2011. The results are not sensitive to the way the distribution of $\theta$ is initialized because the objects of interest are related to earnings changes as opposed to levels. I initialize to a 7.5 percent unemployment rate, which is the value reported by the BLS for January 1977.

Step 6. Compute the moments: aggregate the quarterly earnings observations to annual observations, take 1-year, 3-year, and 5-year changes in log earnings. I use the following moments for each year and for each of the 1-year, 3-year and 5-year changes: the median, and the 10th and 90th percentiles. I express the 10th and 90th percentiles relative to the median (i.e. $50 - 10$ and $90 - 50$). Doing so implies that any differences between the simulated and empirical medians do not change the targets for the widths of the upper and lower tails.

Step 7. Compute the objective function: I take the difference between the simulated moment and the empirical moment from Table A13 in Guvenen et al. (2013). The differences are expressed as squared percentage differences except for the difference in medians, which is expressed relative to the 90th percentile as in Guvenen et al. (2013).

Step 8. Adjust the guess in step 3 and repeat to minimize the objective function from step 7.

As an additional check on the calibrated income process, I compute the standard deviations of the income changes and compared those to the results in Guvenen et al. (2013). Figure 6 shows that the simulated standard deviations are only slightly cyclical while those in the data are more or less acyclical. The simulated standard deviations are somewhat below the observed values.
C Equilibrium conditions

Due to the progressive tax system, a household with skill $\theta_i$ has income proportional to $e^{(1-\theta_i)}$. Given the formulation of the unemployment insurance scheme, this proportionality holds even for unemployed households. This scaling along with homothetic preferences and permanent shocks to $\theta$ can be exploited to eliminate one state variable. Specifically, use lower case letters to denote household variables relative to $e^{(1-\theta_i)}$:

$$c_i = \frac{C_i}{e^{(1-\theta_i)}}, \quad a_i = \frac{A_i}{e^{(1-\theta_i)}}, \quad k_i' = \frac{K_i'}{e^{(1-\theta_i)}}. $$

The household’s Euler equation and budget constraint are

$$C_{i,t}^{\gamma} \geq \beta_i (1 - \omega) \mathbb{E}_t [R_{t+1} C_{i,t+1}^{\gamma}]$$

$$C_{i,t} + K_{i,t} = RK_{i,t-1} + (1 - \tau) W_t e^{(1-\theta_i)\gamma} [n_{i,t} + b(1-n_{i,t})].$$
and in terms of normalized variables these are

\[ c_{i,t}^{-\gamma} \geq \beta_i(1 - \omega)E_t \left[ e^{-(1-b^y)\eta_{i,t+1}} R_{t+1} c_{i,t+1}^{-\gamma} \right] \]  \hspace{1cm} (A2)

\[ c_{i,t} + k_{i,t} = Rk_{i,t-1} e^{-(1-b^y)\eta_{i,t}} + (1 - \tau)W_t e^{(1-b^y)\xi_{i,t}} [n_{i,t} + b^n(1 - n_{i,t})]. \] \hspace{1cm} (A3)

The remaining equations needed to solve the model are: (1), (2), (3), (5), (6), (8), (9), (10), (11), (12), (13), (14), and (A1). These are 13 equations in the 14 variables \( \mu_{1,t}, \mu_{2,t}, \mu_{3,t}, \bar{\mu}_t, z, u, \lambda, x, \zeta, Q, \tau, W, R, \) and \( \bar{K} \). Closing the model requires determining the aggregate capital stock \( \bar{K} \). Following Krusell and Smith (1998) this is done in two ways. In solving the household’s decision problem, I make use of a forecasting rule

\[ \bar{K}' = h(z, \hat{\lambda}, \hat{u}, \hat{u}_{-1}, x, \bar{K}, Q). \] \hspace{1cm} (A4)

I assume that \( h(\cdot) \) is a complete second-order polynomial. In simulating the model, \( \bar{K} \) is determined according to the household decision rules and the dynamics of the distribution of wealth in line with equation (15). To express (15) in normalized terms, note that equations (A2) and (A3) are independent of the household’s permanent income, \( e^{(1-b^y)\theta} \), and so the decision for \( K' \) will be proportional to this permanent income. In particular the household’s savings policy rule can be expressed as

\[ F(A, \theta, n, S) = e^{(1-b^y)\theta} f(a, n, S). \]

In normalized terms we then have

\[ K' = \sum_n \int_a \int_{\theta} e^{(1-b^y)\theta} f(a, n, S) \Gamma^a(da, d\theta, n), \] \hspace{1cm} (A5)

where \( \Gamma^a \) is the joint distribution of cash on hand, skill and employment status at the beginning of the period.
D Numerical methods

Overview I solve the model using the Krusell-Smith algorithm, which involves solving the household’s problem for a given law of motion for the capital stock and updating this law of motion through simulation and least squares curve fitting. For a given law of motion, I solve the household’s problem using a projection method on a grid that is constructed from simulated data generated by a guess of the model solution in the manner described by Judd et al. (2012). This requires alternating between solving the decision problem given a grid and simulating the solution and updating the grid. The steps of the algorithm are as follows:

1. Guess household decision rules and a forecasting rule for the aggregate capital stock.
2. Simulate the economy and record aggregate states.
3. Use simulated data to construct a grid for the aggregate state space.
4. Solve the household’s decision problem on the grid.
5. Simulate the economy and record aggregate states.
6. Use simulated data to construct a grid for the aggregate state space.
7. If the grid has converged then continue, otherwise return to step 4.
8. Update the forecasting rule with least-squares regression.
9. If the forecasting rule has converged stop, otherwise return to step 4.

Initial guesses A good initial guess is important to the success of this algorithm because a poor guess will lead to a situation in step 5 where the economy is being simulated far from the grid on which the problem was solved. In most cases I have found it sufficient to use the linearized solution for the representative agent model as a starting point. The representative agent’s policy rule can be simulated to provide the data for the initial grid and this policy can also serve as a decent guess for the forecasting rule. The success of this guess is premised
on the difference between the representative agent and incomplete markets economies being limited. This is not the case for the baseline economy and this guess is not sufficient for this case. Instead, I found it necessary to gradually build up an initial guess based on versions of the model that are more similar to the representative agent model. I gradually lowered the rate of time-preference of the less patient group to generate this guess.

**Constructing the grid** See Judd et al. (2012). I target a grid with 45 points. As I explain below, I approximate functions of aggregate states with complete second-order polynomials.\(^{14}\) As the dimension of \(S\) is seven there are 36 terms in these polynomials that will be determined by the value of the function on this grid.

**Solving the household’s problem** I solve the household’s problem using the endogenous grid point method (Carroll, 2006). A household’s decision rule can be written in terms of cash on hand relative to permanent income

\[
F(A, \theta, n, \beta, S) = e^{(1-b^y)\theta} f(a, n, \beta, S).
\]

Even though it is not a state, I have included \(\beta\) among the household’s states in order to be explicit about the different types of households whose decision rules must be solved for. For given values of \(n \in \{0, 1\}, \beta,\) and aggregate state \(S\), I approximate the household’s savings function with a piece-wise linear function of 100 knots with more knots placed at low levels of savings and \(k'_{[j]} = 0\). I fix a grid on end-of-period savings \(k'\) such that

\[
k'_{[j]} = e^{(1-b^y)\theta} \hat{f}(a_{[j]}(n, \beta, S), n, \beta, S),
\]

where \([j]\) indexes grid points and \(a_{[j]}(n, \beta, S)\) is the value of normalized cash on hand (relative to \(e^{(1-b^y)\theta}\)) for which a household with states \((n, \beta, S)\) saves \(k'_{[j]}\). \(n\) and \(\beta\) both take two

\(^{14}\)The use of second-order polynomials should not be confused with a second-order perturbation approximation method. The projection method used here minimizes the residual in the model equations across a grid over the state space as opposed to a perturbation method which uses information from the derivatives of the model equations at a single point in the state space.
discrete values and there are 100 values of $j$ so the algorithm must find 400 functions that map $S$ to particular values of $a_{[j]}(n, \beta, S)$. I approximate each of these functions as a complete second-order polynomial in $S$. As there are more grid points than terms in the polynomials approximating these functions, I update the coefficients of the polynomials by least-squares projection.

To compute expectations with respect to aggregate shocks, I use the monomial rule with $2N$ nodes described by Judd et al. (2012). To compute expectations over idiosyncratic shocks I use Gaussian quadrature. Of particular interest is the $\eta$ shock because this has a time-varying distribution. I use Gaussian quadrature with five points in each tail and three points for the central mixture component. As it is only the means of the distributions that are moving with $x$ and not the variance of the mixture components, I construct fixed quadrature grids for each component and shift their locations according to $x$. For the transitory shock, $\xi$, I use Gaussian quadrature with three points.

**Simulation and updating the law of motion** In solving for the law of motion for the aggregate capital stock, I simulate a panel of 100,000 households for 5,500 quarters and discard the first 500 quarters. When drawing the idiosyncratic shocks I reduce the sampling error by, at each period, requiring the cross-sectional average of idiosyncratic productivities to equal the theoretical value of 1 within both the employed and unemployed groups. Using the simulated aggregate capital stock, I update the law of motion with a least squares regression using the same functional form as for the household decision rules (a complete second-order polynomial in the aggregate state). For computing the moments in Table 4, I simulate a panel of 7.2 million households as described in Footnote 11.

**Accuracy of the law of motion for capital** To assess the accuracy of the law of motion for the capital stock, Figure 7 shows a plot of the capital stock generated from simulating the model and the approximate capital stock generated by repeatedly applying the approximate law of motion for capital.15 This is one sample path of shocks for 1000 quarters and the

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15As den Haan (2010) suggests, the sequence of shocks used to simulate the model for the accuracy shock differ from those used to calculate the approximate law of motion.
discrepancy between the two lines is the forecast error that the agents are making at different horizons. One can see that the discrepancy is small even at forecast horizons of 1000 quarters. The maximum absolute log difference between the two series is 0.0089514 and the mean absolute log difference is 0.0036778. Another commonly-reported accuracy check is the $R^2$ of the one-step ahead forecast, which is 0.999992.

**Accuracy of the policy rules**  There are several sources of error in the approximate solution. First, there is the error introduced by the discrepancy between the forecasting rule and the actual dynamics of the aggregate capital stock. Second, there are errors associated with the projection method that arise between grid points when the function being approximated is not of the same form as the approximating function.

To assess the accuracy of the solution, I calculate unit-free Euler equation errors.\textsuperscript{16} For a given state of the economy, $S$, the distribution of wealth, the capital stock, and exogenous variables are predetermined.

\begin{align*}
\text{Pre-determined and exogenous:} & \quad K, z, u, u_{-1}, \lambda, x, Q, \Gamma.
\end{align*}

\textsuperscript{16}See Judd (1992) for an explanation of this accuracy check and the interpretation of the errors in terms of bounded rationality.
Γ is generated by simulating a panel of households. I then use the computed solution to determine the household decision rules

\[
\text{Approx. solutions: } a_{\bar{j}}(n, \beta, S) \quad \forall j, n, \beta.
\]

Using these policy rules, one can compute the savings of each household and then aggregate to find \( K' \) by integrating against Γ. For a given set of aggregate shocks one can then compute \( S' \) from (9), (10), (11), (12), and (14). Given \( S' \) compute \( a_{\bar{j}}(n, \beta, S') \). The Euler equation error is then

\[
\frac{\beta(1 - \omega)\mathbb{E} \left[ e^{-\gamma(1 - b\nu)n'} R(S')c(a', n', \beta, S')^{-\gamma} \right]^{-1/\gamma}}{c(a, n, \beta, S)} - 1
\]

where \( a' = R(S')k'e^{-(1 - b\nu)n'} + (1 - \tau(S'))W(S')e^{(1 - b\nu)\xi'[n' + b\nu(1 - n')]} \) and the consumption functions satisfy \( c + k' = a \). Here \( \mathbb{E} \) represents an expectation over aggregate and idiosyncratic shocks. For aggregate shocks, I use Gaussian quadrature with seven points in each dimension. For idiosyncratic shocks I use the same Gaussian quadrature methods as used to solve the model. For households who are borrowing constrained the Euler equation should not hold. For these households consumption is determined from the borrowing constraint and there is no Euler equation error.

Using these steps, I can compute the Euler equation error for a household with a particular set of states (aggregate and idiosyncratic). To choose a set of aggregate states at which to evaluate the errors, I simulate the economy for 1000 starting from the risky steady state and repeat this 100 times for different sets of random shocks. This produces 100 points that can be considered as draws from the model’s ergodic distribution over the aggregate state space. For idiosyncratic states, I construct a fine grid on normalized cash on hand. Specifically, I use 1000 equally spaced points from 1/1000 to 1000.

Figure 8 summarizes the errors across points in the state space. Each of the panels corresponds to a set of discrete states for a household with the top row showing less patient households and the bottom row more patient and the left column unemployed and the right column employed. Each panel plots the mean and maximum absolute errors across the 100
Figure 8: Euler equation errors. Left column: unemployed; right column: employed; top row: less patient; bottom row: more patient. Maximum and mean across 100 aggregate states.

aggregate states that were tested.

Solving for the policy rule under complete markets  For the complete markets model I use the algorithm described in Judd (1992) that iterates on the Euler equation. I again use a complete second-order polynomial for the savings policy rule.

E Complete markets model

This appendix derives the representative agent Euler equation from the environment presented in Section 2 augmented with a complete set of contingent securities. I assume that trade takes place at an initial period prior to date 0 before any uncertainty has been resolved. I also assume that all households have the same rate of time-preference. Like Shell (1971),
I assume that all current and future generations meet and trade in this initial period. Let $I_{i,t}$ take the value 1 if household $i$ is alive in period $t$ and zero if it is not. I will treat birth and death as random events against which the household can insure. Specifically, let $s^t$ be a history of stochastic events up to date $t$ the probability of which is $\pi_t(s^t)$. These stochastic events dictate the evolution of all idiosyncratic as well as aggregate developments. Let $p_t(s^t)$ be the date-0 price of a unit of the final good at date $t$ and history $s^t$. The household’s utility function is

$$
\sum_{t=0}^{\infty} \sum_{s^t} \beta^t \pi_t(s^t) \frac{C_{i,t}^{1-\gamma}}{1-\gamma} I_{i,t}(s^t).
$$

Notice that a household only values consumption when it is living. In order to prevent them from choosing negative values of consumption when not living I impose $C_{i,t}(s^t) \geq 0$ for all $i$, $t$, and $s^t$. The household’s present-value budget constraint is

$$
\sum_{t=0}^{\infty} \sum_{s^t} p_t(s^t) \left[ C_{i,t}(s^t) + K_{i,t+1}(s^t) - W_t(s^t)\ell_{i,t}(s^t) - \tilde{R}_t(s^t)K_{i,t}(s^{t-1}) \right],
$$

where $\ell_{i,t}$ is the household’s endowment of efficiency units of labor. I assume that all households are identical when trade occurs and each is endowed with an equal share of the initial capital stock.

The household’s Lagrangian is

$$
\mathcal{L} = \sum_{t=0}^{\infty} \sum_{s^t} \beta^t \pi_t(s^t) \frac{C_{i,t}^{1-\gamma}}{1-\gamma} I_{i,t}(s^t)
$$

$$
- \Xi_i \sum_{t=0}^{\infty} \sum_{s^t} p_t(s^t) \left[ C_{i,t}(s^t) + K_{i,t+1}(s^t) - W_t(s^t)\ell_{i,t}(s^t) - \tilde{R}_t(s^t)K_{i,t}(s^{t-1}) \right]
$$

$$
+ \sum_{t=0}^{\infty} \sum_{s^t} \beta^t \pi_t(s^t) \psi_{i,t}(s^t)C_{i,t}(s^t),
$$

where $\Xi_i$ and $\psi_{i,t}(s^t)$ are Lagrange multipliers. The first order condition with respect to consumption is

$$
\beta^t \pi_t(s^t) \left[ C_{i,t}(s^t)^{1-\gamma} I_{i,t}(s^t) + \psi_{i,t}(s^t) \right] = p_t(s^t)\Xi_i.
$$

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The complementary slackness condition is $\psi_{i,t}(s^t)C_{i,t}(s^t) = 0$. By symmetry, the Lagrange multiplier $\Xi_i$ is common across households. It follows that $C_{i,t}(s^t)^{-\gamma}I_{i,t}(s^t) + \psi_{i,t}(s^t)$ must be common across households at a particular date and history. Suppose the household is living, then consumption is positive and common across living households and $\psi_{i,t}(s^t) = 0$. If the household is not living then $\psi_{i,t}(s^t)$ takes the value of the marginal utility of consumption for living households and consumption is zero. This establishes that all living households consume the same amount regardless of their labor income history or age. Define $\bar{C}_t(s^t)$ as the common level of consumption for living households and note $\bar{C}_t(s^t)^{-\gamma} = C_{i,t}(s^t)^{-\gamma}I_{i,t}(s^t) + \psi_{i,t}(s^t)$.

The first order condition with respect to $K_{i,t+1}(s^t)$ is

$$\Xi_i p_t(s^t) = \sum_{s^{t+1}|s^t} \Xi_i p_{t+1}(s^{t+1}) \tilde{R}_{t+1}(s^{t+1}).$$

Substituting for $\Xi_i p_t(s^t)$ from above yields

\[
C_{i,t}(s^t)^{-\gamma}I_{i,t}(s^t) + \psi_{i,t}(s^t)
= \beta \sum_{s^{t+1}|s^t} \frac{\pi_t(s^{t+1})}{\pi_t(s^t)} \left[ C_{i,t+1}(s^{t+1})^{-\gamma}I_{i,t+1}(s^{t+1}) + \psi_{i,t+1}(s^{t+1}) \right] \tilde{R}_{t+1}(s^{t+1})
\]

\[
\bar{C}_t(s^t)^{-\gamma} = \beta \sum_{s^{t+1}|s^t} e^{q_{t+1}(s^{t+1})} \frac{\pi_t(s^{t+1})}{\pi_t(s^t)} \bar{C}_{t+1}(s^{t+1})^{-\gamma} \tilde{R}_{t+1}(s^{t+1})
\]

$$\bar{C}^{-\gamma} = \beta \mathbb{E}_t \left[ C^{t-\gamma} \tilde{R} \right].$$