The role of automatic stabilizers in the U.S. business cycle*

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Abstract

Most countries have automatic rules in their tax-and-transfer systems that are partly intended to stabilize economic fluctuations. This paper measures their effect on the dynamics of the business cycle. We put forward a model that merges the standard incomplete-markets model of consumption and inequality with the new Keynesian model of nominal rigidities and business cycles, and that includes most of the main potential stabilizers in the U.S. data and the theoretical channels by which they may work. We find that the conventional argument that stabilizing disposable income will stabilize aggregate demand plays a negligible role in the dynamics of the business cycle, whereas tax-and-transfer programs that affect inequality and social insurance can have a larger effect on aggregate volatility. However, as currently designed, the set of stabilizers in place in the U.S. has had little effect on the volatility of aggregate output fluctuations or on their welfare costs despite stabilizing aggregate consumption. The stabilizers have a more important role when monetary policy is constrained by the zero lower bound, and they affect welfare significantly through the provision of social insurance.


Keywords: Countercyclical fiscal policy; Heterogeneous agents; Fiscal multipliers.

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1 Introduction

The fiscal stabilizers are the rules in law that make fiscal revenues and outlays relative to total income change with the business cycle. They are large, estimated by the Congressional Budget Office (2013) to account for $386 of the $1089 billion U.S. deficit in 2012, and much research has been devoted to measuring them using either microsimulations (e.g. Auerbach, 2009) or time-series aggregate regressions (e.g. Fedelino et al., 2005). Unlike the controversial topic of discretionary fiscal stimulus, these built-in responses of the tax-and-transfer system have been praised over time by many economists as well as policy institutions.¹ The IMF (Baunsgaard and Symansky, 2009; Spilimbergo et al., 2010) recommends that countries enhance the scope of these fiscal tools as a way to reduce macroeconomic volatility. In spite of this enthusiasm. Blanchard (2006) noted that: “very little work has been done on automatic stabilization [...] in the last 20 years” and Blanchard et al. (2010) argued that designing better automatic stabilizers was one of the most promising routes for better macroeconomic policy.

This paper asks the question: are the automatic stabilizers effective at reducing the volatility of macroeconomic fluctuations? More concretely, we propose a business-cycle model that captures the most important channels through which the automatic stabilizers may attenuate the business cycle, we calibrate it to U.S. data, and we use it to measure their quantitative importance. Our first and main contribution is a set of estimates of how much higher the volatility of aggregate activity would be if some or all of the fiscal stabilizers were removed.

Our second contribution is to investigate the theoretical channels by which the stabilizers may attenuate the business cycle and to quantify their relative importance. The literature suggests four main channels. The dominant mechanism, present in almost all policy discussions of the stabilizers, is the disposable income channel (Brown, 1955). If a fiscal instrument, like an income tax, reduces the fluctuations in disposable income, it will make consumption and investment more stable, thereby stabilizing aggregate demand. In the presence of nominal rigidities, this will stabilize the business cycle. A second channel for potential stabilization works through marginal incentives (Christiano, 1984). For example, with a progressive personal income tax, the tax rate facing workers rises in booms and falls in recessions, therefore encouraging intertemporal substitution of work effort away from booms

and into recessions. Third, automatic stabilizers have a *redistribution* channel. Blinder (1975) argued that if those that receive funds have higher propensities to spend them than those who give the funds, aggregate consumption and demand will rise with redistribution. Oh and Reis (2012) argued that if the receivers are at a corner solution with respect to their choice of hours to work, while the payers work more to offset their fall in income, aggregate labor supply will rise with redistribution. Even if aggregate disposable income and marginal tax rates were held constant, the distribution of this income can affect aggregate demand and marginal incentives and thereby stabilize economic activity. Related is the *social insurance* (or wealth distribution) channel: these policies alter the risks households face with consequences for precautionary savings and the distribution of wealth (Floden, 2001; Alonso-Ortiz and Rogerson, 2010; Challe and Ragot, 2015). For instance, a generous safety net will reduce precautionary savings making it more likely that agents face liquidity constraints after an aggregate shock.

Our third contribution is methodological. We merge the standard incomplete-markets model surveyed in Heathcote et al. (2009) with the standard sticky-price model of business cycles in Woodford (2003). Building on work by Reiter (2009), we show how to numerically solve for the ergodic distribution of the endogenous aggregate variables in a model where the distribution of wealth is a state variable and prices are sticky. This allows us to compute second moments for the economy, and to investigate counterfactuals in which some or all of the stabilizers are not present. We hope that future work will build on this contribution to study the interaction between inequality, business cycles and macroeconomic policy in the presence of nominal rigidities.

We find that our model is able to generate a large fraction of people with low wealth and high marginal propensities to consume, as well as to mimic the variability and cyclicality of the major macroeconomic aggregates and fiscal revenues and spending programs. While the model can generate large multipliers in response to fiscal shocks, we find that the automatic stabilizers have played a minor role in the business cycle. While the variability of aggregate consumption is lower with the stabilizers, the variance of output or hours would actually fall if the stabilizers were eliminated. The usual argument that automatic stabilizers operate through the stabilization of aggregate demand is not borne out by our analysis.

At the same time, we find that the redistribution and social insurance channels are powerful, so that programs that rely on them like food stamps can be effective at reducing the volatility of aggregate output. Moreover, the ineffectiveness of the automatic stabilizers depends on how monetary policy is conducted. If monetary policy is far from optimal, either
due to bad policy or due to the zero lower bound on nominal interest rates binding, then automatic stabilizers can play an important role in aggregate stabilization.

According to our model, scaling back the automatic stabilizers would result in a large drop in a utilitarian measure of social welfare. However, this is mostly due to the redistribution across different groups that these policies induce, and to the social insurance that they provide. Business cycles do not play a large role in the welfare analysis. We do not calculate optimal policy in our model, partly because this is computationally infeasible at this point, and partly because that is not the spirit of our exercise. Our calculations are instead in the tradition of Summers (1981) and Auerbach and Kotlikoff (1987). Like them, we propose a model that fits the US data and then change the tax-and-transfer system within the model to make positive counterfactual predictions on the business cycle.

Literature Review

This paper is part of a revival of interest in fiscal policy in macroeconomics. Most of this literature has focussed on fiscal multipliers that measure the response of aggregate variables to discretionary shocks to policy. Instead, we measure the effect of fiscal rules on the ergodic variance of aggregate variables. This leads us to devote more attention to taxes and government transfers, whereas the previous literature has tended to focus on government purchases.

Focussing on stabilizers, there is an older literature discussing their effectiveness (e.g. Musgrave and Miller, 1948), but little work using modern intertemporal models. Christiano (1984) and Cohen and Follette (2000) use a consumption-smoothing model, Gali (1994) uses a simple RBC model, Andrés and Doménech (2006) use a new Keynesian model, and Hairault et al. (1997) use a few small-scale DSGEs. However, they typically consider the effect of a single automatic stabilizer, the income tax, whereas we comprehensively evaluate several of them to provide a quantitative assessment of the stabilizers as a group. Christiano and Harrison (1999), Guo and Lansing (1998) and Dromel and Pintus (2008) ask whether progressive income taxes change the region of determinacy of equilibrium, whereas we use a model with a unique equilibrium, and focus on the impact of a wider set of stabilizers on the volatility of endogenous variables at this equilibrium. Jones (2002) calculates the effect

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2 For a survey, see the symposium in the Journal of Economic Literature, with contributions by Parker (2011), Ramey (2011) and Taylor (2011).

3 In the United States in 2011, total government purchases were 2.7 trillion dollars. Government transfers amounted to almost as much, at 2.5 trillion. Focussing on the cyclical components, during the 2007-09 recession, which saw the largest increase in total spending as a ratio of GDP since the Korean war, 3/4 of that increase was in transfers spending (Oh and Reis, 2012), with the remaining 1/4 in government purchases.
of estimated fiscal rules on the business cycle using a representative-agent model, whereas we focus on the rules that make up for automatic stabilization and find that heterogeneity is crucial to understand their effects. Finally, some work (van den Noord, 2000; Barrell and Pina, 2004; Veld et al., 2013) uses large macro simulation models to conduct exercises in the same spirit as ours, but their models are often too complicated to isolate the different channels of stabilization and they typically assume representative agents, shutting off the redistribution and social insurance channels that we will find to be important.

Huntley and Michelangeli (2011) and Kaplan and Violante (2014) are closer to us in the use of optimizing models with heterogeneous agents to study fiscal policy. However, they estimate multipliers to discretionary tax rebates, whereas we estimate the systematic impact on the ergodic variance of the automatic features of the fiscal code. Heathcote (2005) analyzes an economy that is hit by tax shocks and shows that aggregate consumption responds more strongly when markets are incomplete due to the redistribution mechanism. We study instead how the fiscal structure alters the response of the economy to non-fiscal shocks. Floden (2001), Alonso-Ortiz and Rogerson (2010), Horvath and Nolan (2011), and Berriel and Zilberman (2011) focus on the effects of tax and transfer programs on average output, employment, and welfare in a steady state without aggregate shocks. Instead, we focus on business-cycle volatility, so we have aggregate shocks and measure variances.

Methodologically, this paper is part of a recent literature using incomplete-market models with nominal rigidities to study business-cycle questions. Oh and Reis (2012) and Guerrieri and Lorenzoni (2011) were the first to incorporate nominal rigidities into the standard model of incomplete markets. Both of them solved only for the impact of a one-time unexpected aggregate shock, whereas we are able to solve for recurring aggregate dynamics. Gornemann et al. (2014) solve a similar problem to ours, but they focus on the distributional consequences of monetary policy. Ravn and Sterk (2013) use a related model to analyze the interaction of market incompleteness, precautionary savings, aggregate demand, and unemployment risks.

Empirically, Auerbach and Feenberg (2000), Auerbach (2009), and Dolls et al. (2012) use micro-simulations of tax systems to estimate the changes in taxes that follows a 1% increase in aggregate income. A much larger literature (e.g Fatas and Mihov, 2012) has measured automatic stabilizers using macro data, estimating which components of revenue and spending are strongly correlated with the business cycle. Whereas this work focusses on measuring the presence of stabilizers, our goal is instead to judge their effect on the business cycle.
2 A business-cycle model with automatic stabilizers

To quantitatively evaluate the role of automatic stabilizers, we would like to have a model that satisfies three requirements.

First, the model must include the four channels of stabilization that we discussed. We accomplish this by proposing a model that includes: (i) intertemporal substitution, so that marginal incentives matter, (ii) nominal rigidities, so that aggregate demand plays a role in fluctuations, (iii) liquidity constraints and unemployment, so that Ricardian equivalence does not hold and there is heterogeneity in marginal propensities to consume and willingness to work, and (iv) incomplete insurance markets and precautionary savings, so that social insurance affects the response to aggregate shocks.

Second, we would like to have a model that is close to existing frameworks that are known to capture the main features of the U.S. business cycle. With complete insurance markets, our model is similar to the neoclassical-synthesis DSGE models used for business cycles, as in Christiano et al. (2005), but augmented with a series of taxes and transfers. With incomplete insurance markets, our model is similar to the one in Krusell and Smith (1998), but including nominal rigidities and many taxes and transfers.

Third and finally, the model must include the main automatic stabilizers present in the data. Table 1 provides an overview of the main components of spending and revenue in the integrated U.S. government budget. Appendix A provides more details on how we define each category.

The first category on the revenue side is the classic automatic stabilizer, the personal income tax system. Because it is progressive in the United States, its revenue falls by more than income during a recession. Moreover, it lowers the volatility of after-tax income, it changes marginal returns from working over the cycle, it redistributes from high to low-income households, and it provides insurance. Therefore, it works through all of the four theoretical channels. We consider three more stabilizers on the revenue side: corporate income taxes, property taxes and sales and excise taxes. All of them lower the volatility of after tax income and so may potentially be stabilizing. Because they have, approximately, a fixed statutory rate, we will refer to them as a group as proportional taxes.4

On the spending side, we consider two stabilizers working through transfers. Unemployment benefits greatly increase in every recession as the number of unemployed rises. Safety-net programs include food stamps, cash assistance to the very poor, and transfers to.

4Average effective corporate income tax rates are in fact countercyclical in the data, mostly as result of recurrent changes in investment tax credits during recessions that are not automatic.
Table 1: The automatic stabilizers in the U.S. government budget

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<thead>
<tr>
<th>Revenues</th>
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<td><strong>Progressive income taxes</strong></td>
<td><strong>Transfers</strong></td>
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<td>Personal Income Taxes</td>
<td>Unemployment benefits</td>
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<td>Safety net programs</td>
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<td><strong>Proportional taxes</strong></td>
<td><strong>Budget deficits</strong></td>
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<td>Corporate Income Taxes</td>
<td>Public deficit</td>
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<td></td>
<td>Government purchases</td>
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<td>Property Taxes</td>
<td>Net interest income</td>
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<tr>
<td>Sales and excise taxes</td>
<td></td>
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<tr>
<td><strong>Budget deficits</strong></td>
<td><strong>Out of the model</strong></td>
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<td>Public deficit</td>
<td>Payroll taxes</td>
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<td></td>
<td>Customs taxes</td>
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<tr>
<td></td>
<td>Licenses, fines, fees</td>
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<tr>
<td><strong>Out of the model</strong></td>
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Notes: Each cell shows the average of a component of the budget as a ratio of GDP, 1988-2007

the disabled. During recessions, more households have incomes that qualify them for these programs and the aggregate quantity of transfers increases.

Interacting with all the previous stabilizers is the *budget deficit*, or the automatic constraint imposed by the government budget constraint. This includes both how fast debt is paid down as well as the fiscal tools used to reduce deficits. We will consider different rules, especially with regards to how government purchases adjust. The convention in the literature measuring automatic stabilizers is to exclude government purchases because there is no automatic rule dictating their adjustment.\(^5\) We will consider both this case, as well as an alternative where government purchases serve as a stabilizer by responding to budget deficits.

The last rows of table 1 include the fiscal programs that we will exclude from our study because they conflict with at least one of our desired model properties. Licenses and fines have no obvious stabilization role. We leave out international flows so that we stay within the standard closed-economy business-cycle model. More important in their size in the budget, we omit retirement, both in its expenses and in the payroll taxes that finance it, and we omit health benefits through Medicare and Medicaid. We exclude them for two

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\(^5\)See Perotti (2005) and Girouard and André (2005) for two of many examples. That literature distinguishes between the built-in stabilizers that respond automatically, by law, to current economic conditions, and the feedback rule that captures the behavior of fiscal authorities in response to current and past information.
complementary reasons. First, so that we follow the convention, since the vast literature on measuring automatic stabilizers to assess structural deficits almost never includes health and retirement spending. Second, because conventional business-cycle models typically ignore the life-cycle considerations that dominate choices of retirement and health spending. The share of the government’s budget devoted to health and retirement spending has been steadily increasing over the years so exploring possible effects of these types of spending on the business cycle is a priority for future work.

The model that follows is the simplest that we could write—and it is already quite complicated—that satisfies these three requirements and includes all of these stabilizers. To make the presentation easier, we will discuss several agents, so that we can introduce one automatic stabilizer per type of agent, but most of them could be centralized into a single household and a single firm without changing the equilibrium of the model.

2.1 Patient households and the personal income tax

We assume that the economy is populated by two groups of households. The first group is relatively more patient and has access to a complete set of insurance markets in which they can insure all idiosyncratic risks. This is not a strong assumption since these agents enjoy significant wealth and would be close to self-insuring, even without state-contingent financial assets. We can then talk of a representative patient household, whose preferences are:

\[ E_0 \sum_{t=0}^{\infty} \beta^t \left[ \log c_t - \psi_1 n_t^{1+\psi_2} \right], \]

where \( c_t \) is consumption and \( n_t \) are hours worked, both non-negative. The parameters \( \beta, \psi_1 \) and \( \psi_2 \) measure the discount factor, the relative willingness to work, and the Frisch elasticity of labor supply, respectively. We assume that there is a unit mass of patient households.

The budget constraint of the representative patient households is

\[ \hat{p}_t c_t + b_{t+1} - b_t = p_t [x_t - \bar{\tau}(x_t) + T^p_t]. \]

The left-hand side has the uses of funds: consumption at the price \( \hat{p}_t \), which includes con-

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6 Even the increase in medical assistance to the poor during recessions is questionable: for instance, in 2007-09 the proportional increase in spending with Medicaid was as high as that with Medicare.

7 We have experimented with simple ways of incorporating these parts of the government budget such as including a payroll tax and treating the outlays on health care and retirement as government purchases. Our results are little affected by these changes.
sumption taxes, plus saving in risk-less bonds \( b_t \) in nominal units. The right-hand side has after-tax income, where \( x_t \) is the real pre-tax income, \( \bar{\tau}^x(x_t) \) are personal income taxes, and \( p_t \) is the price of a unit of final goods. \( T_t^p \) refers to lump-sum transfers, which we will calibrate to zero, but will be useful later to discuss counterfactuals.

The real income of the representative patient household is

\[
x_t = (I_{t-1}/p_t)b_t + d_t + w_t\bar{s}n_t. \tag{3}
\]

It equals the sum of the returns on bonds at nominal rate \( I_{t-1} \), dividends \( d_t \) from owning firms, and wage income. The wage rate is the product of the average wage in the economy, \( w_t \), and the agent’s productivity \( \bar{s} \). This productivity could be an average of the individual-specific productivities of all patient households, since these idiosyncratic draws are perfectly insured.

The patient households own two types of assets explicitly in the model. They trade bonds with the impatient households and the government and they invest capital in the production firms via a holding company that we discuss below. This capital investment is financed by a negative dividend in their budget constraint. In addition, omitted from the model to conserve on notation, the patient households trade Arrow-Debreu securities among themselves to pool their idiosyncratic risks.\(^8\)

The first automatic stabilizer in the model is the personal income tax system. It satisfies:

\[
\bar{\tau}^x(x) = \int_0^x \tau^x(x')dx', \tag{4}
\]

where \( \tau^x : \mathbb{R}^+ \to [0, 1] \) is the marginal tax rate that varies with real income. The system is progressive because \( \tau^x(\cdot) \) is weakly increasing.

### 2.2 Impatient households and transfers

There is a measure \( \nu \) of impatient households indexed by \( i \in [0, \nu] \), so that an individual variable, say consumption, will be denoted by \( c_t(i) \). They have the same period felicity function as patient households, but they are more impatient: \( \hat{\beta} \leq \beta \). Following Krusell and Smith (1998), having heterogeneous discount factors allows us to match the very skewed wealth distribution that we observe in the data. We link this wealth inequality to participation in

\(^8\)The securities that these households trade within themselves to insure against idiosyncratic risks net out to zero and so disappear in the budget constraint of the representative patient household.
financial markets to match the well-known fact that most U.S. households do not directly own any equity (Mankiw and Zeldes, 1991). We assume that the impatient households do not own shares in the firms or own the capital stock. However, their savings can be used to finance capital accumulation by lending to the patient households through the bond market.

Individual impatient households choose consumption, hours of work, and bond holdings \( \{c_t(i), n_t(i), b_{t+1}(i)\} \) to maximize:

\[
E_0 \sum_{t=0}^{\infty} \beta^t \left[ \log c_t(i) - \psi_1 n_t(i)^{1+\psi_2} \right].
\]

(5)

Also like patient households, impatient households can save using risk-free nominal bonds, and pay personal income taxes, so their budget constraint is:

\[
p_t c_t(i) + b_{t+1}(i) - b_t(i) = p_t \left[ x_t(i) - \bar{\pi} x_t(i) + T^s_t(i) \right],
\]

together with a borrowing constraint, \( b_{t+1}(i) \geq 0 \). The lower bound equals the natural debt limit if households cannot borrow against future government transfers.

Unlike patient households, impatient households face two sources of uninsurable idiosyncratic risk: on their labor-force status, \( e_t(i) \), and on their skill, \( s_t(i) \). If a household is employed, then \( e_t(i) = 2 \), and she can choose how many hours to work. While working, her labor income is \( s_t(i) w_t n_t(i) \). The shocks \( s_t(i) \) captures shocks to the worker’s productivity. They generate a cross-sectional distribution of labor income. With some probability, the worker loses her job, in which case \( e_t(i) = 1 \) and labor income is zero. However, now the household collects unemployment benefits \( T^u_t(i) \), which are taxable in the United States. Once unemployed, the household can either find a job with some probability, or exhaust her benefits and qualify for poverty benefits. This is the last state, and for lack of better terms, we refer to their members as the needy or the long-term unemployed. If \( e_t(i) = 0 \), labor income is zero but the household collects food stamps and other safety-net transfers, \( T^s_t(i) \), which are non-taxable. Households in this labor market state are less likely than the unemployed to regain employment. The transition probabilities across labor force states are exogenous, but time-varying.

Collecting all of these cases, the taxable real income of an impatient household is:

\[
x_t(i) = \begin{cases} 
\frac{I_{t-1} b_t(i)}{p_t} + w_t s_t(i) n_t(i) & \text{if employed;} \\
\frac{I_{t-1} b_t(i)}{p_t} + T^u_t(i) & \text{if unemployed;} \\
\frac{I_{t-1} b_t(i)}{p_t} & \text{if needy.}
\end{cases}
\]

(7)
There are two new automatic stabilizers at play in the impatient household problem. First, the household can collect unemployment benefits, $T^u_t(i)$ which equal:

$$T^u_t(i) = \bar{T}^u \min \{s_t(i), \bar{s}^u\}.$$  \hspace{1cm} (8)

Making the benefits depend on the current skill-level captures the link between unemployment benefits and previous earnings, and relies on the persistence of $s_t(i)$ to achieve this. As is approximately the case in the U.S. law, we keep this relation linear with slope $\bar{T}^u$ and a maximum cap $\bar{s}^u$.

The second stabilizer is the safety-net payment $T^s_t(i)$ paid to needy households, which equals:

$$T^s_t(i) = \bar{T}^s.$$  \hspace{1cm} (9)

We assume that these transfers are lump-sum, providing a minimum living standard. In the data, transfers are means-tested, but in our model these families only receive interest income from holding bonds and this is a small amount for most households. When we impose a maximum income cap to be eligible for these benefits, we find that almost no household hits this cap. For simplicity, we keep the transfer lump-sum.

### 2.3 Final goods’ producers and the sales tax

A competitive sector for final goods combines intermediate goods according to the production function:

$$y_t = \left( \int_0^1 y_t(j)^{1/\mu_t} dj \right)^{\mu_t},$$  \hspace{1cm} (10)

where $y_t(j)$ is the input of the $j^{th}$ intermediate input. There are shocks to the elasticity of substitution across intermediates that generate exogenous movements in desired markups, $\mu_t > 1$.

The representative firm in this sector takes as given the final-goods pre-tax price $p_t$, and pays $p_t(j)$ for each of its inputs. Cost minimization together with zero profits imply that:

$$y_t(j) = \left( \frac{p_t(j)}{p_t} \right)^{\mu_t/(1-\mu_t)} y_t,$$  \hspace{1cm} (11)

$$p_t = \left( \int_0^1 p_t(j)^{1/(1-\mu_t)} dj \right)^{1-\mu_t}.$$  \hspace{1cm} (12)

Goods purchased for consumption are taxed at the rate $\tau^c$, so the after-tax price of con-
This consumption tax is our next automatic stabilizer, as it makes actual consumption of goods a fraction \(1/(1 + \tau^c)\) of pre-tax spending on them.

### 2.4 Intermediate goods and corporate income taxes

There is a unit continuum of intermediate-goods monopolistic firms, each producing variety \(j\) using a production function:

\[
y_t(j) = a_t k_t(j)^\alpha \ell_t(j)^{1-\alpha},
\]

where \(a_t\) is productivity, \(k_t(j)\) is capital used, and \(\ell_t(j)\) is effective labor.

The labor market clearing condition is

\[
\int_0^1 \ell_t(j) dj = \int_0^\nu s_t(i)n_t(i) di + \bar{s}n_t.
\]

The demand for labor on the left-hand side comes from the intermediate firms. The supply on the right-hand side comes from employed households, adjusted for their productivity.

The firm maximizes after-tax nominal profits

\[
d_t(j) \equiv (1 - \tau^k) \left[ \frac{p_t(j)}{p_t} y_t(j) - w_t \ell_t(j) - (\nu r_t + \delta) k_t(j) - \xi \right] - (1 - \nu) r_t k_t(j),
\]

taking into account the demand function in equation (11). The firm’s costs are the wage bill to workers, the rental of capital at rate \(r_t\) plus depreciation of a share \(\delta\) of the capital used, and a fixed cost \(\xi\). The parameter \(\nu\) measures the share of capital expenses that can be deducted from the corporate income tax. In the United States, dividends and capital gains pay different taxes. While this distinction is important to understand the capital structure of firms and the choice of retaining earnings, it is immaterial for the simple firms that we just described.\(^9\)

Intermediate firms set prices subject to nominal rigidities a la Calvo (1983) with probability of price revision \(\theta\). Since they are owned by the patient households, they use their

\(^9\)Another issue is the treatment of taxable losses (Devereux and Fuest, 2009). Because of carry-forward and backward rules in the U.S. tax system, these should not have a large effect on the effective tax rate faced by firms, although firms do not seem to claim most of these tax benefits. We were unable to find a satisfactory way to include these considerations into our model without greatly complicating the analysis.
stochastic discount factor, $\lambda_{t,t+s}$, to choose price $p_t(j)^*$ at a revision date with the aim of maximizing expected future profits:

$$E_t \left[ \sum_{s=0}^{\infty} (1-\theta)^s \lambda_{t,t+s} d_{t+s}(j) \right] \quad \text{subject to: } p_{t+s}(j) = p_t(j)^*. \quad (17)$$

The new automatic stabilizer is the corporate income tax, which is a flat rate $\tau^k$.

### 2.5 Capital-goods firms and property income taxes

A representative firm owns the capital stock and rents it to the intermediate-goods firms, taking $r_t$ as given. If $k_t$ denotes the capital held by this firm, then the market for capital clears when:

$$k_t = \int_0^1 k_t(j) dj. \quad (18)$$

This firm invests in new capital $\Delta k_{t+1} = k_{t+1} - k_t$ subject to adjustment costs to maximize after-tax profits:

$$d^k_t = r_t k_t - \Delta k_{t+1} - \frac{\zeta}{2} \left( \frac{\Delta k_{t+1}}{k_t} \right)^2 k_t - \tau^p v_t. \quad (19)$$

The value of this firm, which owns the capital stock, is then given by the recursion:

$$v_t = d^k_t + E_t (\lambda_{t,t+1} v_{t+1}).$$

The new automatic stabilizer, the property tax, is a fixed tax rate $\tau^p$ that applies to the value of the only property in the model, the capital stock. A few steps of algebra show the conventional results from the $q$-theory of investment:

$$v_t = q_t k_t, \quad (20)$$

$$q_t = 1 + \zeta \left( \frac{\Delta k_{t+1}}{k_t} \right). \quad (21)$$

Because, from the second equation, the price of the capital stock is procyclical, so will property values, making the property tax a potential automatic stabilizer.

Finally, note that total dividends sent to patient households, $d_t$, come from every intermediate firm and the capital-goods firm:

$$d_t = \int_0^1 d^*_t(j) dj + d^k_t. \quad (22)$$
We do not include investment tax credits. They are small in the data and, when used to attenuate the business cycle, they have been enacted as part of stimulus packages, not as automatic rules.

2.6 The government budget and deficits

The government budget constraint is:

\[
p_t \left[ \tau^c \left( \int_0^\nu c_t(i)di + c_t \right) + \tau^p q_t k_t + \int_0^\nu \bar{\tau}^x(x_t(i))di + \bar{\tau}^x(x_t) + \tau^k \left( \int_0^1 \hat{d}(j) dj + (1 - v)r_t k_t \right) - \int_0^\nu [T_t^n(i) + T_t^p(i)] di \right] = p_t g_t + I_{t-1} B_t + B_t - B_{t+1} + p_t T_t^p. \tag{23}
\]

On the left-hand side are all of the automatic stabilizers discussed so far: sales taxes, property taxes and personal income taxes in the first line, and corporate income taxes and transfers in the second line.\(^{10}\) On the right-hand side are government purchases, \(g_t\) and government bonds \(B_t\). The market for bonds will clear when:

\[
B_t = \int_0^\nu b_t(i)di + b_t. \tag{24}
\]

In steady state, the stabilizers on the left-hand side imply a positive surplus, which is offset by steady-state government purchases \(\bar{g}\). Since we set transfers to the patient households in the steady state to zero, \(\bar{T}^p = 0\), the budget constraint then determines a steady state amount of debt \(\bar{B}\), which is consistent with the government not being able to run a Ponzi scheme.

Outside of the steady state, as outlays rise and revenues fall during recessions, the left-hand side of equation (23) decreases leading to an automatic increase in the budget deficit during recessions. We study the stabilizing properties of deficits in terms how fast and with what tool the debt is paid.

We assume that the lump-sum tax on the patient households and government purchases adjust to close deficits because they are the fiscal tools that least interfere with the other stabilizers. They do not affect marginal returns as do the distortionary tax rates, and they do not have an important effect on the wealth and income distribution as do transfers to impatient households. We assume simple linear rules similar to the ones estimated by Leeper\(^{10}\).

\(^{10}\)\(\hat{d}(j)\) are taxable profits, the term in brackets on the right-hand side of equation (16).
et al. (2010):

$$\log(g_t) = \log(\bar{g}) - \gamma^G \log \left( \frac{B_t/p_t}{\bar{B}} \right), \quad (25)$$

$$T^p_t = \bar{T}^p + \gamma^T \log \left( \frac{B_t/p_t}{\bar{B}} \right). \quad (26)$$

The parameters $\gamma^G, \gamma^T > 0$ measure the speed at which the deficits from recessions are paid over time. Large values of these parameters imply deficits are paid right away the following period; if they are close to zero, they take arbitrarily long to get paid. Their relative size determines the relative weight that purchases and taxes have on fiscal stabilizations.

### 2.7 Shocks and business cycles

In our baseline, monetary policy follows a simple Taylor rule:

$$I_t = \bar{I} + \phi \Delta \log(p_t) - \varepsilon_t, \quad (27)$$

with $\phi > 1$. We omitted the usual term in the output gap for two reasons. First, because with incomplete markets, it is no longer clear how to define a constrained-welfare natural level of output to which policy should respond. Second, because it is known that in this class of models with complete markets, a Taylor rule with an output term is quantitatively close to achieving the first best. We preferred to err on the side of having an inferior monetary policy rule so as to raise the likelihood that fiscal policy may be effective. We will consider alternative monetary policy rules in section 5.

Three aggregate shocks hit the economy: technology, $\log(a_t)$, monetary policy, $\varepsilon_t$, and markups, $\log(\mu_t)$. Therefore, both aggregate-demand and aggregate-supply shocks may drive business cycles, and fluctuations may be efficient or inefficient. We assume that all shocks follow independent AR(1) processes for simplicity.\(^{11}\) It would be straightforward to include trend growth in the model, but we leave it out since it plays no role in the analysis.

The idiosyncratic shocks to households, $e_t(i)$ and $s_t(i)$ are first-order Markov processes. Moreover, the transition matrix of labor-force status, the three-by-three matrix $\Pi_t$, depends on a linear combination of the aggregate shocks. In this way, we let unemployment vary with the business cycle to match Okun’s law.

\(^{11}\)We have also experimented with including investment-specific technology shocks and found similar results. More details are available from the authors.
2.8 Equilibrium

An equilibrium in this economy is a collection of aggregate quantities \((y_t, k_t, d_t, v_t, c_t, n_t, b_{t+1}, x_t, d^k_t);\) aggregate prices \((p_t, \hat{p}_t, w_t, q_t);\) impatient household decision rules \((c_t(b, s, e), n_t(b, s, e));\) a distribution of households over assets, skill levels, and employment statuses; individual firm variables \((y_t(j), p_t(j), k_t(j), l_t(j), d_t(j));\) and government choices \((B_t, I_t, g_t)\) such that:

(i) patient households maximize (1) subject to the budget constraint (2)-(3),
(ii) the impatient household decision rules maximize (5) subject to (6)-(7),
(iii) the distribution of households over assets, skill, and employment levels evolves in a manner consistent with the decision rules and the exogenous idiosyncratic shocks,
(iv) final-goods firms behave optimally according to equations (11)-(13),
(v) intermediate-goods firms maximize (17) subject to (11), (14), (16),
(vi) capital-goods firms maximize expression (19) so their value is given by (20)-(21),
(vii) fiscal policy respects (23) and (25)-(26) while monetary policy follows (27),
(viii) markets clear for labor in equation (15), for capital in equation (18), for dividends in equation (22) and for bonds in equation (24).

Appendix C derives the optimality conditions that we use to solve the model. We evaluate the mean and variance of aggregate endogenous variables in the ergodic distribution of the equilibrium in this economy.

3 The positive properties of the model

The model just laid out combines the uninsurable idiosyncratic risk familiar from the literature on incomplete markets with the nominal rigidities commonly used in the literature on business cycles. Our first contribution is to show how to solve this general class of models, and to briefly discuss some of their properties.

3.1 Solution algorithm

Our full model is challenging to analyze because the solution method must keep track not only of aggregate state variables, but also of the distribution of wealth across agents. One candidate algorithm is the Krusell and Smith (1998) algorithm, which summarizes the distribution of wealth with a few moments of the distribution. We opt instead for the solution algorithm developed by Reiter (2009), because this method can be easily applied to models with a rich structure at the aggregate level, including a large number of aggregate state
variables. Here we give an overview of the solution algorithm, while Appendix E provides more details.

The Reiter algorithm first approximates the distribution of wealth with a histogram that has a large number of bins. The mass of households in each bin becomes a state variable of the model. The algorithm then approximates the household decision rules with a discrete approximation, a spline. In this way, the model is converted from one that has infinite-dimensional objects to one that has a large, but finite, number of variables.

Using standard techniques, one can find the stationary competitive equilibrium of this economy in which there is idiosyncratic uncertainty, but no aggregate shocks. Reiter (2009) calls for linearizing the model with respect to aggregate states, and solving for the dynamics of the economy as a perturbation around the stationary equilibrium without aggregate shocks using existing linear rational expectations algorithms. The resulting solution is non-linear with respect to the idiosyncratic variables, but linear with respect to the aggregate states.\(^{12}\)

Approximation errors arise both because the projection method to solve the Euler equation involves some approximation error between grid points, and because of the linearization with respect to aggregate states. To assess the accuracy of the solution, we compute Euler-equation errors and report the results in Appendix F.

### 3.2 Calibrating the model

We calibrate as many parameters as possible to the properties of the automatic stabilizers in the data. For government spending and revenues our target data is in table 1, which reflects the period 1988-2007. For macroeconomic aggregates, we use quarterly data over a longer period, 1960-2011, so that we can include more recessions in the sample and periods outside the Great Moderation so as not to underestimate the amplitude of the business cycle.\(^{13}\)

For the three proportional taxes, we use parameters related to preferences or technology to match the tax base in the NIPA accounts, and choose the tax rate to match the average revenue reported in table 1, following the strategy of Mendoza et al. (1994). The top panel of table 2 shows the parameter values and the respective targets.

For the personal income tax, we followed Auerbach and Feenberg (2000) and calculated

---

\(^{12}\) The method proposed by Reiter (2010) allows for a finer discretization of the distribution of wealth by using techniques from linear systems theory to compress the state of the model. We have used this to verify that our results are not affected by adopting a finer discretization of the distribution of wealth.

\(^{13}\) To ensure that the government’s budget balances in steady state we scale the outlays that we target in our calibration up by 1.024 so that total revenues and outlays are equal in table 1. For example, we calibrate total safety net spending to be 1.04% of GDP as opposed to 1.02% as appears in table 1.
Table 2: Calibration of the parameters

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Parameter Description</th>
<th>Value</th>
<th>Target (Source)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Panel A. Tax bases and rates</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\tau_c$</td>
<td>Tax rate on consumption</td>
<td>0.0535</td>
<td>Avg. revenue from sales taxes (Table 1)</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Discount factor of patient households</td>
<td>0.989</td>
<td>Consumption-income ratio = 0.689 (NIPA)</td>
</tr>
<tr>
<td>$\tau_p$</td>
<td>Tax rate on property</td>
<td>0.00258</td>
<td>Avg. revenue from property taxes (Table 1)</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Coefficient on labor in production</td>
<td>0.296</td>
<td>Capital income share = 0.36 (NIPA)</td>
</tr>
<tr>
<td>$\tau_k$</td>
<td>Tax rate on corporate income</td>
<td>0.35</td>
<td>Statutory rate</td>
</tr>
<tr>
<td>$\nu$</td>
<td>Deduction of capital costs</td>
<td>0.68</td>
<td>Avg. revenue from corporate income tax (Table 1)</td>
</tr>
<tr>
<td>$\xi$</td>
<td>Fixed costs of production</td>
<td>0.575</td>
<td>Corporate profits / GDP = 9.13% (NIPA)</td>
</tr>
<tr>
<td>$\mu$</td>
<td>Desired gross markup</td>
<td>1.1</td>
<td>Avg. U.S. markup (Basu, Fernald, 1997)</td>
</tr>
<tr>
<td>Panel B. Government outlays and debt</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\bar{T}_u$</td>
<td>Unemployment benefits</td>
<td>0.144</td>
<td>Avg. outlays on unemp. benefits (Table 1)</td>
</tr>
<tr>
<td>$\tilde{s}_u/\bar{T}_u$</td>
<td>Max. UI benefit / avg. income</td>
<td>0.66</td>
<td>Typical state law (BLS, 2008)</td>
</tr>
<tr>
<td>$\bar{T}_s$</td>
<td>Safety-net transfers</td>
<td>0.151</td>
<td>Avg. outlays on safety-net benefits (Table 1)</td>
</tr>
<tr>
<td>$G/Y$</td>
<td>Steady-state purchases / output</td>
<td>0.145</td>
<td>Avg. outlays on purchases (Table 1)</td>
</tr>
<tr>
<td>$\gamma^T$</td>
<td>Fiscal adjustment speed (tax)</td>
<td>-1.6</td>
<td>St. dev. of deficit/GDP = 0.0093 (NIPA)*</td>
</tr>
<tr>
<td>$\gamma^G$</td>
<td>Fiscal adjustment speed (spending)</td>
<td>-1.28</td>
<td>St. dev. of log spending = 0.0126 (NIPA)*</td>
</tr>
<tr>
<td>$B/Y$</td>
<td>Steady-state debt / output</td>
<td>1.7</td>
<td>Avg. interest expenses (Table 1)</td>
</tr>
<tr>
<td>Panel C. Income and wealth distribution</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\nu$</td>
<td>Impatient households per patient household</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>$\hat{\beta}$</td>
<td>Discount factor of imp. households</td>
<td>0.979</td>
<td>Wealth of top 20% by wealth</td>
</tr>
<tr>
<td>$\bar{s}$</td>
<td>Skill level of pat. households</td>
<td>3.72</td>
<td>Income of top 20% by wealth (SCF)</td>
</tr>
<tr>
<td>Panel D. Business-cycle parameters: externally calibrated</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\theta$</td>
<td>Calvo price stickiness</td>
<td>0.286</td>
<td>Avg. price spell duration = 3.5 (Klenow, Malin, 2011)</td>
</tr>
<tr>
<td>$\psi_1$</td>
<td>Labor supply</td>
<td>21.6</td>
<td>Avg. hours worked = 0.31 (Cooley, Prescott, 1995)</td>
</tr>
<tr>
<td>$\psi_2$</td>
<td>Labor supply</td>
<td>1</td>
<td>Frisch elasticity = 1/2 (Chetty, 2012)</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Depreciation rate</td>
<td>0.0114</td>
<td>Annual depreciation expenses / GDP = 0.046 (NIPA)</td>
</tr>
<tr>
<td>$\rho_p$</td>
<td>Autocorrelation markup shock</td>
<td>0.85</td>
<td></td>
</tr>
<tr>
<td>Panel E. Business-cycle parameters: internally calibrated</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\zeta$</td>
<td>Adjustment costs for investment</td>
<td>6</td>
<td>St. dev. of $I = 0.0531$ (NIPA)*</td>
</tr>
<tr>
<td>$\rho_z$</td>
<td>Autocorrelation productivity shock</td>
<td>0.75</td>
<td>Autocorrel. of log GDP = 0.864 (NIPA)*</td>
</tr>
<tr>
<td>$\sigma_z$</td>
<td>St. dev. of productivity shock</td>
<td>0.00294</td>
<td>St. dev. of log GDP = 0.0154 (NIPA)*</td>
</tr>
<tr>
<td>$\rho_m$</td>
<td>Autocorrelation monetary shock</td>
<td>0.62</td>
<td>Largest AR for inflation = 0.85 (Pivetta, Reis, 2006)</td>
</tr>
<tr>
<td>$\sigma_m$</td>
<td>St. dev. of monetary shock</td>
<td>0.00353</td>
<td>Share of output variance due to shock = 0.25</td>
</tr>
<tr>
<td>$\sigma_p$</td>
<td>St. dev. of markup shock</td>
<td>0.0251</td>
<td>Share of output variance due to shock = 0.25</td>
</tr>
<tr>
<td>$\phi_p$</td>
<td>Interest-rate rule on inflation</td>
<td>1.55</td>
<td>St. dev. of inflation = 0.638 (NIPA)</td>
</tr>
</tbody>
</table>

* Indicates HP filtered data using smoothing parameter 1600 for quarterly data.
federal and state taxes for a typical household using TAXSIM. We averaged the tax rates across states weighted by population, and across years between 1988 and 2007. We then fit a cubic function of income to the resulting schedule, and splined it with a flat line above a certain level of income so that the fitted function would be non-decreasing. The result is in figure 1. The cubic-linear schedule approximates the actual taxes well, and its smoothness is useful for the numerical analysis. We then added an intercept to this schedule to fit the effective average tax rate. This way, we made sure we fitted both the progressivity of the tax system (via TAXSIM) and the average tax rates (via the intercept).

Panel B calibrates the parameters related to government spending. Both parameters governing transfer payments are set to match the average outlays from these programs, while the cap on unemployment benefits uses an approximation of existing law.

Panel C contains parameters that relate to the distribution of income and wealth across households. According to the Survey of Consumer Finances, 83.4% of the wealth is held by the top 20% in the United States (Díaz-Giménez et al., 2011). We then picked the discount factor of the impatient households to match this target.

Omitted from the table for brevity, but available in Appendix B, are the Markov transition matrices for skill level and employment. We used a 3-point grid for household skill
levels, which we constructed from data on wages in the Panel Study for Income Dynamics. The transition matrix across employment status varies linearly with a weighted average of the three aggregate shocks to match the correlation between employment and output. We set its parameters to match the flows in and out of the two main government transfer programs, food stamps and unemployment benefits, both on average and over the business cycle.

Finally, Panels D and E have all the remaining parameters. Most are standard, but a few deserve some explanation. First, the Frisch elasticity of labor supply plays an important role in many intertemporal business-cycle models. Consistent with our focus on taxes and spending, we use the value suggested in the recent survey by Chetty (2012) on the response of hours worked to several tax and benefit changes. We have found that the results on the impact of automatic stabilizers on business cycle volatilities are not very sensitive to this parameter although the impact of taxes on the average level of activity is clearly sensitive to the choice of labor supply elasticity. Second, we choose the variance of monetary shocks and markup shocks so that a variance decomposition of output attributes them each 25% of aggregate fluctuations. There is great uncertainty on the empirical estimates of the sources of business cycles, but this number is not out of line with some of the estimates in the literature. Our results turn out to not be sensitive to these choices.

Whereas the parameters in panel D are set directly to match the target moments, those in panel E (together with \( \hat{\beta} \) and \( \bar{s} \) in panel C) are determined jointly in an internal calibration of the model’s ergodic distribution, that estimates these 9 parameters to minimize the distance to the 9 target moments. While we have tried to use data to discipline our choices of parameters as much as possible, there is nevertheless uncertainty surrounding many of the values reported in table 2. A formal estimation and characterization of this uncertainty is beyond the scope of this study.

### 3.3 Optimal behavior and equilibrium inequality

Figure 2 uses a simple diagram to describe the stationary equilibrium of the model without aggregate shocks. For the sake of clarity, the figure depicts an environment in which there are no taxes that distort saving decisions.

The downward-sloping curve is the demand for capital, with slope determined by diminishing marginal returns. The supply of savings by patient households is perfectly elastic at the inverse of their time-preference rate just as in the neoclassical growth model. Because they are the sole holders of capital, the equilibrium capital stock in the model is determined by the intersection of these two curves. Introducing taxes on capital income, like the per-
personal or corporate income taxes, raises the pre-tax return on savings that patient households require and lowers the equilibrium capital stock.

If impatient households were also fully insured, their supply of savings would be the horizontal line at \( \hat{\beta}^{-1} \). But, because of the idiosyncratic risk they face, they have a precautionary saving motive. Therefore, they are willing to hold bonds at lower interest rates. Their aggregate savings are given by the upward-sloping curve. Because in the steady state without aggregate shocks, bonds and capital must yield the same return, equilibrium bond holdings by impatient households are given by the point to the left of the equilibrium capital stock. The difference between the total amount of government bonds outstanding and those held by impatient households gives the bond holdings of patient households.

Figure 3 shows the optimal savings decisions of impatient households at each of the employment states. When households are employed, they save, so the policy function is above the 45° line. When they do not have a job, they run down their assets. As wealth reaches zero, those out of a job consume all of their safety-net income, leading to the horizontal segment along the horizontal axis in their savings policies.

Figure 4 shows the ergodic wealth distribution for impatient households. Two features of these distributions will play a role in our results. First, many needy households hold essentially no assets, so they live hand to mouth. Second, the figure shows a counterfactual wealth distribution if the two transfer programs are significantly cut. Because not being
Figure 3: Optimal savings policies

Table 3: Fraction of sub-population with low wealth.

<table>
<thead>
<tr>
<th>Employment (e)</th>
<th>Share of population</th>
<th>Skill level (s)</th>
<th>Low</th>
<th>Medium</th>
<th>High</th>
</tr>
</thead>
<tbody>
<tr>
<td>Employed</td>
<td>0.692</td>
<td>0.574</td>
<td>0.072</td>
<td>0.017</td>
<td></td>
</tr>
<tr>
<td>Unemployed</td>
<td>0.021</td>
<td>0.589</td>
<td>0.080</td>
<td>0.016</td>
<td></td>
</tr>
<tr>
<td>Needy</td>
<td>0.087</td>
<td>0.769</td>
<td>0.486</td>
<td>0.334</td>
<td></td>
</tr>
</tbody>
</table>

Low wealth is defined as assets less than the average quarterly income for an employed household with the same skill level.

employed now leads to a larger loss of income, households save more, which raises their wealth in all states. Table 3 shows the proportion of each skill-employment group that has assets less than one quarter’s average income for an employed individual with the same skill level.

3.4 Business cycles

Before we use this model to perform counterfactuals on the effect of the automatic stabilizers on the business cycle, we inspect whether it can mimic the key features of U.S. business cycles.

Figure 5 shows the impulse responses to the three aggregate shocks, with impulses equal to one standard deviation. The model captures the positive co-movement of output, hours
and consumption, as well as the hump-shaped responses of hours to a TFP shock. Inflation rises with expansionary monetary shocks, but falls with productivity and markup shocks. As usual in the standard Calvo model, the responses are fairly short-lived. In spite of all the heterogeneity, the aggregate responses to shocks are similar to those of the standard new neoclassical-synthesis model in Woodford (2003) and Christiano et al. (2005) that has been widely used to study business cycles in the past decade.

Turning to the unconditional moments of the business cycle, we chose the parameters of our model so that it mimics the standard deviations of output, unemployment and inflation. Therefore, the model already matches the unconditional second moments in these variables. Also by calibration, the model already reproduces the main features of the wealth and income distribution.

The marginal propensity to consume (MPC) has received a great deal of attention in the study of fiscal policy and it also plays an important role in our model. All else equal, a larger MPC would raise the strength of the disposable-income channel as any fluctuation in disposable income would translate into a larger movement in aggregate demand. Moreover, with more heterogeneous MPCs, the redistribution channel will be stronger as moving resources from agents with higher to lower MPCs will have a larger impact on aggregate demand.

Table 4 shows the distribution of MPCs in our economy according to employment status.
Figure 5: Impulse responses to the aggregate shocks
and wealth percentile. Parker et al. (2011) use tax rebates to estimate an average MPC between 0.12 and 0.3. Our model is able to generate MPCs that go from 0.02 to 0.49, so that both in the spread and on average, it has the potential to give these two channels a strong role. Among the needy and the low-skill unemployed, the MPCs are quite large and more individuals enter these groups in a recession. Comparing Tables 3 and 4 it is clear that the groups with high MPCs are those with few assets.

### 3.5 The effects and cyclicality of fiscal policy

Our calibration strategy targeted the average revenue generated from each tax. A test of the model is whether it can also match the cyclicality of these revenues. Table 5 reports the covariance of revenues and outlays with detrended output.\(^{14}\) For most spending and tax categories the model-predicted cyclicalities are not only of the right sign but also quite close to their empirical counterparts. The main failure is that the model generates counter-cyclical revenues for the corporate income tax while these are strongly pro-cyclical in the data. The reason is that our model, like any new Keynesian model, has countercyclical markups. Therefore, because corporate profits are strongly linked to markups, the revenue

\(^{14}\)Detrending is important because the structure of the government budget has changed significantly across decades, with some sources of revenue and spending growing fast and others declining. We use the HP filter to calculate trend output, and divide both fiscal revenues and outlays by trend output before calculating the covariance with detrended output. Because the cyclical component of GDP is stationary by construction, by calculating the covariance, we are not letting the trends in fiscal items affect the estimates. Moreover, by detrending all variables in the government budget constraint by the common output trend, the covariances of all of the terms in equation 23 have to add up.
Table 5: Covariance with detrended GDP.

<table>
<thead>
<tr>
<th>Fiscal variable</th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tax revenues</td>
<td>0.095</td>
<td>0.044</td>
</tr>
<tr>
<td>Sales tax</td>
<td>0.004</td>
<td>0.007</td>
</tr>
<tr>
<td>Property tax</td>
<td>-0.002</td>
<td>0.003</td>
</tr>
<tr>
<td>Personal income tax</td>
<td>0.052</td>
<td>0.046</td>
</tr>
<tr>
<td>Corporate income tax</td>
<td>0.041</td>
<td>-0.013</td>
</tr>
<tr>
<td>Purchases</td>
<td>-0.009</td>
<td>0.022</td>
</tr>
<tr>
<td>UI payments</td>
<td>-0.020</td>
<td>-0.010</td>
</tr>
<tr>
<td>Net government savings</td>
<td>0.185</td>
<td>0.136</td>
</tr>
</tbody>
</table>

Quarterly data from 1960:I - 2011:IV and expressed relative to potential output (HP filter trend).

from taxing corporate income is countercyclical in the model, even though it is procyclical in the data. Overall, the discrepancy between the predicted and actual cyclicality in total tax revenues is 0.051, which is almost entirely explained by the discrepancy in the cyclicality of corporate income tax revenue (0.054).

A simple extension of the model can eliminate this gap with little change to its relevant properties. If only a fraction of the fixed operating cost $\xi$ is deductible from the corporate income tax and this fraction is counter-cyclical, then we can exactly match the cyclicality of the corporate income tax revenues. As the fixed cost is not a choice variable, its tax treatment does not change marginal incentives, so the dynamics of the model barely change. Moreover, we can partially defend this admittedly ad hoc assumption with the limited deductibility of corporate income tax credits.

Figure 6 shows the impulse responses of output to shocks to three fiscal variables: an increase in government purchases, a cut in the personal income tax, and a redistribution of wealth from patient households to the needy. In the first two cases we change one parameter of the model unexpectedly and only at date 1, and trace out the aggregate dynamics as the economy converges back to its old ergodic distribution. In the third case, we redistribute wealth at date 1 and simulate the model starting from that new distribution towards the ergodic case. In each case, we normalize the response of output by the size of the policy change measured in terms of its impact on the government budget. The response to redistribution is non-linear in the size of the transfer, which we set so that each needy household receives one percent of average household income.

Because these shocks have no persistence, their aggregate effect will always be limited.
Yet, we find that they induce relatively large changes in output. Calculating multipliers as the ratio of the change in output to the change in the deficit over the first year of the experiment, we find reasonably-sized numbers: 0.90 for purchases, 0.27 for taxes, and 0.23 for redistribution. These are larger than the typical response in the neoclassical-synthesis model. With household heterogeneity, the aggregate demand effects of these fiscal policies are larger, since the MPC of the needy in particular are very high, and the aggregate supply effect is larger as well, since the employed households bear more of the financing of fiscal expansions, so they are particularly encouraged to work harder when marginal taxes fall or their total after-tax wealth falls. Our model is therefore able to generate significant effects of fiscal policy.

Figure 7 shows the same responses when we modify the utility function to have no wealth effects as in Greenwood et al. (1988). Qualitatively, the responses of output are similar. Quantitatively, the impact of government purchases is larger, since government purchases raise aggregate demand by more with these preferences, while the impact of redistribution is smaller, since the employed households no longer choose to work hard as a result of being taxed more heavily. This confirms our intuition on which economic channels are at work in the model, and provides motivation to consider the quantitative effect that this change will have on our estimates of the role of the stabilizers.
Figure 7: Impulse responses to three fiscal experiments with no wealth effect on labor supply

3.6 Two special cases

In the analysis that follows, we consider two special cases of our model as benchmarks that help isolate different stabilization channels. First, with complete markets, households can diversify idiosyncratic risks to their income. The following assumption eliminates these risks:

**Assumption 1.** All households trade a full set of Arrow securities, so they are fully insured, and they are equally patient, $\hat{\beta} = \beta$.

It will not come as a surprise that if this assumptions holds, there is a representative agent in this economy. More interesting, the problem she solves is familiar:

**Proposition 1.** Under assumption 1, there is a representative agent with preferences:

$$\max \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ \log(c_t) - (1 + E_t) \psi_1 \frac{n_t^{1+\psi_2}}{1 + \psi_2} \right\},$$

$$27$$
and with the following constraints:

\[
\hat{p}_t c_t + b_{t+1} - b_t = p_t [x_t - \bar{\tau}(x_t) + T^n_t],
\]

\[
x_t = \frac{I_{t-1}}{p_t} b_t + w_t s_t (1 + E_t) n_t + d_t + T^n_t,
\]

\[
s_t = \left[ \frac{1}{1 + E_t} s_t^{1+1/\psi_2} + \frac{E_t}{1 + E_t} \int_0^{\nu} s_t^{1+1/\psi_2} dt \right]^{1+1/\psi_2},
\]

where \(1 + E_t\) is total employment, including patient and impatient households and \(T^n_t\) is net non-taxable transfers to the household.

The proof is in Appendix D. With the exception of the exogenous shocks to employment, the problem of this representative agent is fairly standard. Moreover, on the firm side, optimal behavior by the goods-producing firms leads to a new Keynesian Phillips curve, while optimal behavior by the capital-goods firm produces a familiar IS equation. Therefore, with complete markets, our model is of the standard neoclassical synthesis variety (Woodford, 2003) that has been intensively used to study business cycles over the past decade.

The complete-markets case is useful, not just because it is familiar, but also because it allows us to study the effectiveness of automatic stabilizers when distributional issues are set aside. In this version of the model, the marginal incentives and the disposable income channels are the only two mechanisms at work.

A second special case that we will consider replaces the impatient household’s optimal savings function with the assumption that all impatient households live hand-to-mouth. That is, they consume all of their after-tax income at every date and hold zero bonds. This can be seen as a limit when \(\hat{\beta}\) approaches zero. It is inspired by the savers-spenders model of Mankiw (2000). In this case, a measure of 80% of all consumers behave as if they were at the borrowing constraint, with an MPC of 1.

This benchmark is useful for several reasons. First, because it is close to the ultra-Keynesian model in Gali et al. (2007) that combines hand-to-mouth behavior with nominal rigidities to be able to generate a positive multiplier of government purchases on private consumption. For the study of fiscal policy, this is one of the closest models to the IS-LM benchmark that is at the center of policy debates on fiscal policy. Second, the assumption of hand-to-mouth behavior raises the marginal propensity to consume by brute force.\(^{15}\) A large

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\(^{15}\)Heathcote (2005) and Kaplan and Violante (2014) raise the MPC in a more elegant way by, respectively, lowering the discount factor and introducing illiquid assets, but these are hard to accomplish in our model while simultaneously keeping it tractable and able to fit the business-cycle facts and the wealth and income distributions.
MPC, here literally equal to one for the impatient households, maximizes the strength of the disposable income channel. Third, in the hand-to-mouth model, there are no precautionary savings so the social insurance channel is shut off. Our model potentially overstates the role of precautionary savings as households are infinitely lived and therefore have plenty of time to accumulate assets. Compared to our full model, the hand-to-mouth alternative is therefore useful to isolate the channels at work.

4 The effect of automatic stabilizers on the business cycle

To assess whether automatic stabilizers alter the dynamics of the business cycle, we calculate the fraction by which the variance of aggregate activity would increase if we removed some, or all, of the automatic stabilizers. If \( V \) is the ergodic variance at the calibrated parameters, and \( V' \) is the variance at the counterfactual with some of the stabilizers shut off, we define, following Smyth (1966), the stabilization coefficient:

\[
S = \frac{V'}{V} - 1.
\]

This differs from the measure of “built-in flexibility” introduced by Pechman (1973), which equals the ratio of changes in taxes to changes in before-tax income, and is widely used in the public finance literature.\(^{16}\) Whereas built-in flexibility measures whether there are automatic stabilizers, our goal is instead to estimate whether they are effective at reducing the volatility of aggregate quantities.

To best understand the difference, consider the following result, proven in Appendix D:

**Proposition 2.** If assumption 1 holds, so there is a representative agent, and:

1. the Calvo probability of price adjustment \( \theta = 1 \), so prices are flexible;
2. the personal income tax is proportional, so \( \tau^x(\cdot) \) is constant;
3. the probability of being employed is constant over time;

\(^{16}\)See Dolls et al. (2012) for a recent example, and an attempt to go from built-in flexibility to stabilization, by making the strong assumption that aggregate demand equals output and that poor households have MPCs of 1 while rich households have MPCs of zero.
4. there are infinite adjustments costs, $\gamma \to +\infty$, and no depreciation, $\delta = 0$, so capital is fixed;

5. there are no fixed costs of production, $\xi = 0$;

then the variance of the log of output is equal to the variance of the log of productivity and $S = 0$.

While this result and the assumptions supporting it are extreme, it serves a useful purpose. While assumption 1 shuts off the redistribution and social insurance channels of stabilization, the other assumptions in proposition 2 switch off the aggregate demand channel, since prices are flexible, and the marginal incentives channels, as households and firms face the same marginal taxes in booms and recessions. The result in proposition 2 confirms that, in the absence of these channels, the automatic stabilizers have no effect. Moreover, note that the estimates of the size of the stabilizer following the Pechman (1973) approach would be large in this economy. Yet, the stabilizers in this economy have no impact on the volatility of log output and this is reflected by our version of the Smyth (1966) measure.

We begin by considering the roles of each of the stabilizers separately. In doing so, we set $\gamma^G = 0$ in the fiscal rule so that we show the effect of changing the stabilizers as cleanly as possible without changing the dynamics of government purchases due to the new dynamics for government debt. Because the lump-sum taxes, which are the other means for fiscal adjustment, are approximately neutral, they do not risk confusing the effects of the stabilizers with their financing. We then conduct an experiment of reducing all of the stabilizers at the same time to calculate the total effect of the automatic stabilizers on the business cycle.

### 4.1 The effect of proportional taxes on the business cycle

Table 6 considers the following experiment: we cut the tax rates $\tau^c$, $\tau^p$ and $\tau^k$ each by 10%, and replaced the lost revenue of 0.6% of GDP by a lump-sum tax on the patient households.

Lowering proportional taxes lowers the variance of the business cycle by a negligible amount. In fact, removing the stabilizer, actually leads to a slightly more stable economy. In the hand-to-mouth economy, as expected, consumption is less stable as the variance of after-tax income is higher without the proportional taxes. But even then, the effect on the variance of output is only 1%. At the same time, when these taxes are removed, output and consumption are significantly higher on average in all economies.
Table 6: The effect of proportional taxes on the business cycle.

<table>
<thead>
<tr>
<th></th>
<th>Full model</th>
<th></th>
<th>Representative agent</th>
<th></th>
<th>Hand-to-mouth</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>variance</td>
<td>average</td>
<td>variance</td>
<td>average</td>
<td>variance</td>
<td>average</td>
</tr>
<tr>
<td>output</td>
<td>-0.0100</td>
<td>0.0117</td>
<td>-0.0019</td>
<td>0.0115</td>
<td>0.0105</td>
<td>0.0116</td>
</tr>
<tr>
<td>hours</td>
<td>-0.0005</td>
<td>0.0004</td>
<td>0.0029</td>
<td>0.0015</td>
<td>0.0047</td>
<td>0.0006</td>
</tr>
<tr>
<td>consumption</td>
<td>-0.0098</td>
<td>0.0093</td>
<td>-0.0182</td>
<td>0.0090</td>
<td>0.0400</td>
<td>0.0092</td>
</tr>
</tbody>
</table>

Proportional change caused by cutting the stabilizer.

Table 7: The effect of the level of tax rates on the business cycle.

<table>
<thead>
<tr>
<th></th>
<th>Full model</th>
<th></th>
<th>Representative agent</th>
<th></th>
<th>Hand-to-mouth</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>variance</td>
<td>average</td>
<td>variance</td>
<td>average</td>
<td>variance</td>
<td>average</td>
</tr>
<tr>
<td>output</td>
<td>-0.0051</td>
<td>0.0078</td>
<td>-0.0127</td>
<td>0.0076</td>
<td>-0.0600</td>
<td>0.0075</td>
</tr>
<tr>
<td>hours</td>
<td>-0.0140</td>
<td>0.0036</td>
<td>-0.0090</td>
<td>0.0076</td>
<td>-0.0155</td>
<td>0.0034</td>
</tr>
<tr>
<td>consumption</td>
<td>-0.0203</td>
<td>0.0089</td>
<td>-0.0142</td>
<td>0.0087</td>
<td>-0.0264</td>
<td>0.0086</td>
</tr>
</tbody>
</table>

Proportional change caused by cutting the stabilizer.

Intuitively, a higher tax rate on consumption lowers the returns from working and so lowers labor supply and output on average. However, because the tax rate is the same in good and bad times, it does not induce any intertemporal substitution of hours worked, nor does it change the share of disposable income available in booms versus recessions. Likewise, the taxes on corporate income and property may discourage saving and affect the average capital stock. But they do not do so differentially across different stages of the business cycle and so they have a negligible effect on volatility.

Table 7 instead cuts the intercept in the personal income tax by two percentage points. The conclusions for the full model are similar. Again, no intertemporal trade-offs change and lower taxes actually come with slightly less volatile business cycles. Section 4.3 discusses the mechanism behind this fall in volatility.

4.2 The effect of transfers on the business cycle

To evaluate the impact of our two transfer programs, unemployment and poverty benefits, we reduced spending on both by 0.6% of GDP, the same amount in the experiment on proportional taxes. This is a uniform 80% reduction in the transfers amounts. Again, we replaced the fall in outlays with a lump-sum transfer to the patient households. The results
are in table 8. Transfers have a close-to-zero effect on the average level of output and hours, yet they have a substantial effect on their volatility. Reducing transfer payments raises output volatility by 6% and raises the variance of hours worked by as much as 9%.

Aside from the social-insurance channel, there is also a redistribution channel behind the impact of transfers on aggregate volatility. In a recession, there are more households without a job so more transfers in the aggregate. Transfers have no direct effect on the labor supply of recipients as they do not have a job in the first place. However, they are funded by higher taxes on the patient households, who raise their hours worked in response to the reduction in their wealth. This stabilizes hours worked and output.

At the same time, without transfers, the volatility of aggregate consumption falls by 1%. To understand why, note that the transfers provide social insurance against a major idiosyncratic shock that impatient households face. As households face more risk without transfers, they accumulate more assets. This was visible in figure 4, with the large shift of the wealth distribution to the right when transfers are reduced. With more savings, impatient households are better able to smooth their consumption in response to fluctuations in income caused by aggregate shocks and aggregate consumption becomes more stable.

The two special cases also confirm that redistribution and precautionary savings are behind the effectiveness of transfers. In the representative-agent economy, both of these channels are shut off, and the transfer experiment has a negligible effect on all variables. In the hand-to-mouth economy, eliminating the public insurance provided by transfers raises the volatility of aggregate consumption. This is as expected, since a large fraction of the population does not smooth their consumption. Nonetheless, the volatility of output now slightly falls without transfers. The hand-to-mouth economy maximizes the disposable-income channel since every dollar given to impatient households is spent, raising output because of sticky prices. Yet, we see that, quantitatively, this effect accounts for little of the stabilizing effects of transfers in our economy.

### Table 8: The effect of transfers on the business cycle.

<table>
<thead>
<tr>
<th></th>
<th>Full model</th>
<th></th>
<th>Representative agent</th>
<th></th>
<th>Hand-to-mouth</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>variance</td>
<td>average</td>
<td>variance</td>
<td>average</td>
<td>variance</td>
<td>average</td>
</tr>
<tr>
<td>output</td>
<td>0.0603</td>
<td>-0.0004</td>
<td>-0.0063</td>
<td>0.0002</td>
<td>-0.0083</td>
<td>-0.0042</td>
</tr>
<tr>
<td>hours</td>
<td>0.0944</td>
<td>-0.0098</td>
<td>-0.0037</td>
<td>0.0002</td>
<td>0.0047</td>
<td>-0.0017</td>
</tr>
<tr>
<td>consumption</td>
<td>-0.0133</td>
<td>-0.0004</td>
<td>-0.0119</td>
<td>0.0002</td>
<td>0.1003</td>
<td>-0.0048</td>
</tr>
</tbody>
</table>

Proportional change caused by cutting the stabilizer.
This intuition also suggests that the effectiveness of transfers relies on a positive wealth effect on labor supply. When we repeated the same experiment with preferences without this wealth effect, the variance of output then increases by more, 11.4%, without the stabilizers, while the variance of consumption now increases as well, by 6.6%, in contrast with the results in 8. The intuition is as follows. Under standard preferences, households use labor supply as a form of precautionary insurance. In a recession, the increase in unemployment risk induces them to not only consume less but also to increase labor supply in order to accumulate additional savings. With Greenwood et al. (1988) preferences, the household responds to changes in risk only through consumption, not labor supply. Therefore, consumption and aggregate demand must change by more, and transfers become more effective.\textsuperscript{17}

By taking the unemployment rate to be exogenous, our analysis does not incorporate the impact of aggregate demand stabilization on the extent of idiosyncratic risk. This channel has been studied extensively by Ravn and Sterk (2013). Conversely, by taking the unemployment rate to be exogenous, our analysis does not incorporate the disincentive effect of unemployment benefits on the incentive for unemployed workers to engage in costly search, as in Young (2004), or for workers to accept lower wages when employed, as in Hagedorn et al. (2013).\textsuperscript{18}

### 4.3 The effect of progressive income taxes on the business cycle

Our next experiment replaces the progressive personal income tax with a proportional, or flat, tax that raises the same revenue in steady state. Table 9 has the results.

Progressive income taxes have a modest effect on the volatility of output or hours, but

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\textsuperscript{17}The importance of wealth effects for the effectiveness of transfers has recently been emphasized by Athreya et al. (2014). Yet, there is no empirical consensus on how large this wealth effect is.

\textsuperscript{18}An earlier version of this paper considered an extension of the model that captures the disincentive effects of transfers. Results are available from the authors upon request.
moving to a flat tax would raise the average level of economic activity significantly with output and consumption increasing by 4%. This stands in contrast to our results for transfers, even though both are redistributive policies. To understand this difference, we can consider the four channels we discussed in the introduction.

First, because marginal tax rates rise with income this discourages labor supply and lowers average hours and investment leading to reduced average income. This well-understood mechanism works in the cross-section, discouraging individual households from trying to raise their individual income. However, the level of progressivity in the current U.S. tax system is modest in the sense that the marginal tax rate function is relatively flat above median income—recall figure 1. Therefore, the marginal tax rate that many employed households face changes little between booms and recessions. This induces little substitution over time, and therefore has a negligible effect on the variance.

On average activity, though, the effect is large. With a flat tax, because more tax revenue is collected from households with less income, then the high-income households face a significantly lower marginal tax rate. Therefore, they save more, the average capital stock is higher, and so the impact of flattening the tax system on average income is large.

Second, the redistribution channel is significantly weaker than with transfers, because it is less targeted. When the needy receive transfers they cannot reduce their labor supply any further. In contrast, the personal income tax mostly redistributes among employed households. The recipients lower their labor supply in response to their higher income, and little stabilization results.

The important roles of redistribution and precautionary savings are again highlighted by the two special cases, where these two channels are shut off. The table shows that in either the representative-agent or the hand-to-mouth economies, a flat tax leads instead to significantly less volatile business cycles. Further calculations, that we do not report for brevity, show that this fall in volatility is in large part driven by the joint presence of monetary policy shocks and sticky prices.

To understand what is going on, recall the basic mechanism for why a positive monetary policy shock causes a boom with sticky prices: lower nominal interest rates lead to lower real interest rates, which raises consumption, demand for output, and if prices do not change, then raises hours worked and investment. Now, with a progressive tax, first the after-tax return on saving faced by households, \((1 - \tau^x(x_{t+1}))I_t\), is both lower as well as less sensitive to variations in the nominal interest rate, which are driven by inflation. As a result, the progressive tax makes the after-tax real interest rate respond less strongly to inflation. Second, with a
4.4 The effect of all stabilizers on the business cycle

We now combine all of the experiments above. In the counterfactual, a flat tax replaces the progressive personal income tax, proportional taxes are cut by 10%, and unemployment and poverty benefits are cut by 80%. Finally, we decrease the two fiscal adjustment coefficients proportionately so that the variance of budget deficits falls by 10%. Altogether, we see this as a feasible across-the-board reduction in the scope of the automatic stabilizers.

Table 10 shows the results of the overall experiment in our full model. The main result is in the first two numbers in the table: the stabilizers have had a marginal effect on the volatility of the U.S. business cycle in output or hours. Removing the stabilizers would significantly raise the variance of aggregate consumption because government purchases would not be as cyclical. Moreover, by lowering marginal tax rates, it would be a significantly richer economy on average. Even though we found in the previous experiments that the safety-net transfers, could be quite powerful at reducing the volatility of the business cycle, our results show that the current mix of stabilizers actually increase the volatility of aggregate output and hours.

4.5 The role of debt financing

Both the persistence of budget deficits after recessions and the fiscal instrument used to pay for them matter for the effect of any countercyclical fiscal policy, including the automatic

<table>
<thead>
<tr>
<th></th>
<th>Full model</th>
<th></th>
<th>Representative agent</th>
<th></th>
<th>Hand-to-mouth</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>variance</td>
<td>full model</td>
<td>variance</td>
<td>full model</td>
<td>variance</td>
<td>full model</td>
</tr>
<tr>
<td>output</td>
<td>-0.0229</td>
<td>0.0567</td>
<td>-0.0756</td>
<td>0.0533</td>
<td>-0.1381</td>
<td>0.0557</td>
</tr>
<tr>
<td>hours</td>
<td>-0.0296</td>
<td>0.0344</td>
<td>-0.0399</td>
<td>0.0429</td>
<td>-0.0432</td>
<td>0.0311</td>
</tr>
<tr>
<td>consumption</td>
<td>0.1232</td>
<td>0.0603</td>
<td>0.1833</td>
<td>0.0564</td>
<td>0.1938</td>
<td>0.0593</td>
</tr>
</tbody>
</table>

Proportional change caused by cutting the stabilizer.
Table 11: The role of the fiscal adjustment rule.

<table>
<thead>
<tr>
<th></th>
<th>Baseline</th>
<th>No spending response</th>
<th>No spending response and balanced budget</th>
<th>Distortionary taxes adjust</th>
</tr>
</thead>
<tbody>
<tr>
<td>output</td>
<td>-0.0229</td>
<td>-0.0026</td>
<td>-0.0002</td>
<td>-0.1207</td>
</tr>
<tr>
<td>hours</td>
<td>-0.0296</td>
<td>-0.0193</td>
<td>-0.0184</td>
<td>-0.0363</td>
</tr>
<tr>
<td>consumption</td>
<td>0.1232</td>
<td>-0.0361</td>
<td>-0.0318</td>
<td>0.1544</td>
</tr>
</tbody>
</table>

Proportional change caused by cutting the stabilizer.

stabilizers. To study this role, we repeated the experiment of reducing all automatic stabilizers as in section 4.4, but with alternative assumptions about fiscal adjustment. The results are shown in table 11.

First, we contrasted our baseline economy with an alternative economy where only the lump-sum taxes adjust to close the deficits so \( \gamma^G = 0 \). In this economy, government purchases are constant. The stabilizing effect of the automatic stabilizer on aggregate consumption now disappears. In the baseline, the budget deficit in a recession leads to a reduction in purchases that crowds in private consumption. With \( \gamma^G = 0 \), this no longer happens.

The third column of table 11 shows the effect of not only setting \( \gamma^G \) to zero, but also of raising \( \gamma^T \) to infinity so that the government balances its budget every period. The results are almost identical to the previous experiment. While Ricardian equivalence does not hold in our economy, changing the time profile of the taxes on patient households has a small quantitative effect.

The third experiment replaces the rule for the adjustment lump-sum taxes in equation (26), with a similar rule that adjusts the tax rates on the proportional taxes and the intercept in the progressive tax system. We pick the speed of adjustment so that a given change in the public debt generates the same revenue as in the baseline rule. The fourth column in 11 shows a large destabilizing effect of the stabilizers under this rule. There are three reasons for why raising tax rates in a recession to pay for the debt contracts economic activity in our model. First, the property and corporate income taxes rise discouraging investment. Second, the sales tax rises, and is expected to fall when the economy recovers and the public debt is paid, making households want to consume and work less in the recession. Third, the personal income tax rises and is expected to fall in the future, discouraging labor supply. As a result, now eliminating the stabilizers, and the need for these tax adjustments would actually end up leading to less volatile fluctuations. Since government purchases still adjust, the crowding-in effect on private consumption is still present.
To conclude, changing the timing of deficits per se has little effect on the economy. But the way in which these deficits are financed can have a significant effect on volatility. In particular, raising distortionary taxes in response to public deficits raises the volatility of activity.

5 Monetary policy and automatic stabilizers

Our results show that the automatic stabilizers have overall little effect on the dynamics of the business cycle. In particular, the usual arguments about the benefits of the stabilizers in aggregate demand management are not supported by our findings. There are two ways we could have arrived at this conclusion. One possibility is that the model might attribute little importance to aggregate demand management in general. In this section we argue that this is not the case. Instead, we argue that monetary policy comes close to reaching the first best already so there is little additional role for fiscal policy. We then show that when monetary policy is far from optimal, the automatic stabilizers have an important stabilizing effect.

5.1 The roles of price stickiness and monetary policy

Figure 5 already showed that shocks to monetary policy have a significant effect on output in our economy. According to the model, a 25 basis point unexpected increase in interest rates lowers output on impact by 0.3%. While the aggregate demand channel of fiscal policy is weak, monetary policy still plays a significant role in the economy.

Table 12 repeats the experiment of reducing all the stabilizers in our model under different assumptions about monetary policy. First, a useful benchmark is the case where prices are fully flexible, and it is shown in the second column of the table. This eliminates the role of monetary policy entirely and neutralizes the aggregate demand channel. A common finding in the representative-agent version of our business-cycle model without taxes and transfers is that a finely-tuned monetary policy can come close to reaching the first best (Woodford, 2003). We explore this in the third column of the table, which shows the same experiment with sticky prices but now with a monetary policy rule that Schmitt-Grohé and Uribe (2007) showed is close to optimal in a version of the Christiano et al. (2005) model: \( \log\left(\frac{I_t}{I_{t-1}}\right) = 0.77 \log\left(\frac{I_{t-1}}{I_{t-2}}\right) + 0.75 \log(\pi_{t-1}) + 0.02 \log\left(\frac{y_{t-1}}{y_{t-2}}\right) \). This rule has the virtue of depending only on observables, so it avoids the difficulty of defining the right concept of the output gap. The impact of the stabilizers is very similar to the flexible-price
Table 12: The effect of all stabilizers on the business cycle.

<table>
<thead>
<tr>
<th></th>
<th>Flexible prices</th>
<th>S.G.-U.</th>
<th>Baseline</th>
<th>Output</th>
<th>Aggressive</th>
<th>Accommodative</th>
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<tr>
<td>output</td>
<td>-0.0428</td>
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<td>-0.0229</td>
<td>-0.0333</td>
<td>-0.0339</td>
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</tr>
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<td>hours</td>
<td>-0.0390</td>
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<td>-0.0296</td>
<td>-0.0116</td>
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</tr>
<tr>
<td>consumption</td>
<td>0.1165</td>
<td>0.1256</td>
<td>0.1232</td>
<td>0.0905</td>
<td>0.0898</td>
<td>0.0016</td>
</tr>
<tr>
<td>inflation</td>
<td>-0.4123</td>
<td>-0.2907</td>
<td>-0.2828</td>
<td>0.0822</td>
<td>0.0436</td>
<td>0.5201</td>
</tr>
</tbody>
</table>

Proportional change caused by cutting the stabilizer.
Welfare expressed in consumption equivalents.

This confirms the conjecture that with an effective monetary policy, there is little room left for the automatic stabilizers to work through aggregate demand.

The last three columns of our table consider different versions of a Taylor rule that sets the expected after-tax interest rate as a function of the current inflation rate\(^\text{19}\)

\[
E_t [1 - \tau^*(x_{t+1})] I_t = \bar{I} + \phi_p \Delta \log(p_t) + \phi_y \Delta \log(y_t) - \varepsilon_t. \tag{28}
\]

Focusing monetary policy on the after-tax interest rate makes the analysis more transparent because, as we explained at the end of section 4.3, the progressive income tax interacts with the monetary policy rule to determine the effective response of real interest rates to inflation.\(^\text{20}\) With this after-tax rule, the interpretation of \(\phi_p\) is closer to the more familiar case without taxes on interest income. Varying \(\phi_p\) and \(\phi_y\) lets us study the effect of more aggressive responses of monetary policy to inflation, and of responding to output as well, respectively.

The aggressive policy rule sets \(\phi_p = 1.75\) whereas our baseline has \(\phi_p = 1.55\), both with \(\phi_y = 0\). The output policy rule has \(\phi_p = 1.55\) as in the baseline but now \(\phi_y = 0.125\). In all three cases, the results are similar to the flexible price benchmark.

The accommodative policy rule sets \(\phi_p = 1.03\) so the after-tax real interest rate is quite insensitive to changes in inflation. Under such a rule, demand shocks will lead to larger fluctuations in activity as they are not offset by monetary policy. In contrast to the other cases, here we find that the automatic stabilizers have an important role in stabilizing output and hours. Under the accommodative policy, the stabilizers have little effect on the volatility of

\(^{19}\)We use the expected tax-rate of the patient households in this rule.

\(^{20}\)That a constant tax on interest income alters the effective monetary policy rule was previously noted by Edge and Rudd (2007). In our model this effect is larger due to the progressivity of the tax system.
aggregate consumption. This makes sense as the strong effect of the stabilizers on aggregate consumption in the baseline arose out of changes in the dynamics of government purchases. With an accommodative monetary policy, however, private consumption is insulated from changes in the dynamics of government purchases.

To sum up, the rules that either approximate the U.S. data, or are optimal in a related model, or are particularly aggressive, all seem to effectively manage aggregate demand leaving little room for fiscal policy. But with the accommodative rule, the stabilizers substantially reduce the volatility of aggregate output and hours.

5.2 Automatic stabilizers at the zero lower bound

One situation where monetary policy is very accommodative is when nominal interest rates are at the zero lower bound (ZLB). At the same time, much recent research has shown that fiscal policy can be particularly powerful when nominal interest rates do not respond to inflation (Woodford, 2011), and that different fiscal instruments can have widely different impacts relative to each other and to the case where the Taylor principle holds (Eggertsson, 2011). Given the often paradoxical results that the literature has found at the ZLB, it is not clear how the role of the automatic stabilizers might change.

At the same time, it is not easy to solve our model when the ZLB binds. For one, most algorithms rely on the ZLB being a current state that will never repeat itself in the future (Eggertsson and Woodford, 2003), whereas our focus has been on the ergodic steady state. Second, we have to solve the model non-linearly to capture the important precautionary savings and social insurance channels, but non-linear solution algorithms for economies at the ZLB are still in their infancy and cannot solve models as large as ours (Fernandez-Villaverde et al., 2014). Third, hitting the ZLB and staying there for more than one period is difficult in models with realistic capital adjustment costs, because of the investment boom that comes with low interest rates (Christiano et al., 2011).

To solve these problems, we make the following simplifications. First, we set the capital adjustment cost to infinity so that the capital stock is fixed. Second, and related, we raise the degree of price stickiness to \( \theta = 0.15 \), still in line with some empirical estimates, because this helps ensure the existence of a determinate equilibrium.\(^{21}\) Third, instead of the ergodic distribution, we calculate a perfect foresight transition path starting from a stationary equi-

\(^{21}\) We verified that the results in the paper so far do not change much with this new value for \( \theta \). Increasing price stickiness helps dampen the explosive dynamics of inflation at the zero lower bound by having fewer firms update their prices and by making current inflation depend more heavily on future inflation and less on current marginal costs.
librium at date 0 where there are no aggregate shocks but households still face idiosyncratic uncertainty. We found it necessary to make these simplifications in order to incorporate the strong non-linearities at the ZLB, but this comes at a cost of eliminating investment and aggregate uncertainty both of which could have important consequences for the quantitative results.

At date 1, everyone learns that the rate of time preference of all households falls by 0.25% for 15 periods, a standard shock in the ZLB literature. Moreover, at the same time, the risk of becoming unemployed rises by 1.35% percentage points per quarter and the job-finding rate of needy households falls by 0.89% for 8 quarters, so that we generate a cumulative drop in employment of 4%, close to the peak-to-trough decline in employment during the U.S. Great Recession. Appendix G explains in how we construct an equilibrium transition path.

The solid lines in figure 8 show the dynamics of the nominal interest rate, output, aggregate consumption and the total consumption of impatient households. The zero lower bound binds for the first two periods of the transition before gradually returning to its steady state value. Aggregate consumption drops by 6% percentage points on impact and output by 4%. Impatient households are particularly affected by the deterioration in labor market conditions so their consumption drops by nearly 10% on impact. The dashed lines in the figure show the economy’s response to the same set of shocks when all stabilizers have been reduced as in our baseline experiment. The impact of the shock is now substantially larger. The ZLB binds for a further period, and consumption and output fall by an additional 2% and 1%, respectively. With the reduction of social insurance, impatient households are hurt even more, and their consumption falls by an additional 5% for a total fall of 15%.

These results suggest that the automatic stabilizers are more effective in mitigating the extent of the contraction during a zero lower bound episode. This finding is consistent with other studies of the power of fiscal policy at the zero lower bound (e.g. Christiano et al., 2011).

6 The welfare effects of automatic stabilizers

The automatic stabilizers affect welfare partly due to their impact on the amplitude of the business cycle that we have focused on so far. At the same time, they also affect welfare by changing the average level of activity or the extent of public insurance that they provide against idiosyncratic risks. This section discusses these different effects on welfare to answer

\[ \text{Specifically, the discount factors } \beta \text{ and } \hat{\beta} \text{ both rise by a factor of 1.0025.} \]
Figure 8: The zero lower bound episode with and without automatic stabilizers.
Table 13: Welfare cost to impatient households of reducing automatic stabilizer policies. Change in consumption equivalents.

<table>
<thead>
<tr>
<th>Employment</th>
<th>Skill</th>
<th>Wealth percentile</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>10</td>
</tr>
<tr>
<td>Employed Low</td>
<td>-0.174</td>
<td>-0.173</td>
</tr>
<tr>
<td>Employed Med.</td>
<td>-0.139</td>
<td>-0.136</td>
</tr>
<tr>
<td>Employed High</td>
<td>-0.101</td>
<td>-0.099</td>
</tr>
<tr>
<td>Unemployed Low</td>
<td>-0.253</td>
<td>-0.244</td>
</tr>
<tr>
<td>Unemployed Med.</td>
<td>-0.200</td>
<td>-0.185</td>
</tr>
<tr>
<td>Unemployed High</td>
<td>-0.142</td>
<td>-0.131</td>
</tr>
<tr>
<td>Needy Low</td>
<td>-0.371</td>
<td>-0.371</td>
</tr>
<tr>
<td>Needy Med.</td>
<td>-0.354</td>
<td>-0.354</td>
</tr>
<tr>
<td>Needy High</td>
<td>-0.334</td>
<td>-0.271</td>
</tr>
</tbody>
</table>

whether the stabilizers are desirable from a welfare perspective.

6.1 The overall welfare consequences of automatic stabilizers

Table 13 shows the change in the welfare of different agents in our economy from reducing all the stabilizers as in the experiment in section 4.4. In this calculation we consider taking a household with their current individual state variables from the economy with stabilizers and placing them into the economy without stabilizers. We take both economies to be at their respective steady states but the welfare of the agents reflects the anticipation of fluctuations going forward. In the table, an entry of $-0.10$ indicates that a household has lower welfare without the stabilizers by an amount equivalent to 10% of consumption.

As shown in the table, all impatient households are worse off in the world without stabilizers. The equally-weighted average welfare loss in the table is $-0.151$. The welfare of the patient households, in contrast, increases by 0.136. The stabilizers have a large redistributive component, so a policy of scaling down transfers and moving to a flat tax benefits the rich patient households and hurts the poor impatient households. Across impatient households, it is also clear that the policy change has large redistributive effects. The low-wealth needy lose particularly large amounts as they are entirely reliant on transfers for their current consumption. These disparate effects make it hard to state whether the stabilizers are beneficial or not. One controversial answer is to take a utilitarian social welfare function that weighs each group by their population: in this case cutting the stabilizers lowers average welfare by 0.080 consumption-equivalent units.
The numbers in the table compare ergodic distributions. However, one of the reasons why average output in the economy is higher without stabilizers is that households face higher risk and raise their precautionary savings. These come at the expense of lower consumption in the transition, so ignoring this transition may lead to an under-estimate of the welfare benefits of the stabilizers. Yet, when we calculated the welfare of agents from the moment of the policy change onwards, we found that the welfare estimates are only slightly lower than in table 13.

The welfare changes in the table are large for two complementary reasons. First, because the stabilizers are providing social insurance, which the uninsured impatient households benefit from. Second, because the stabilizers change the average level of post-tax income among different types of households, redistributing resources even in the absence of shocks.

### 6.2 Isolating the impact of recessions

Another difficulty with assessing the effect of the stabilizers in welfare is that most of their benefits have little to do with the business cycle. Social insurance may be desirable and useful, regardless of whether it leads to business cycles that are more, less, or similarly volatile. To investigate this, we now look at the effect of the stabilizers during a recession.

In particular, we compute the welfare cost of a series of recessions with and without the stabilizers. We consider four recessionary episodes: a two standard deviation drop in TFP, a two standard deviation contractionary monetary policy shock, a two standard deviation inflationary markup shock, and the zero lower bound episode described in section 5.2. For each, we compute a perfect foresight transition for 250 quarters and the utility of agents at the moment the shock is realized. We then compare this utility to the case without shocks in which the economy will remain at the steady state. This gives us the welfare cost of these recessions. Table 14 shows the results.

With any of the shocks that we consider for our baseline model, the cost of business cycles is small. As noted by Lucas (1987), this is a general feature of business cycle models.
and ours is no different. The exception is the ZLB, where the costs are close to 3% of consumption.

More interesting, without the stabilizers, there is a large increase in the welfare cost of the recessions in relative terms. Without stabilizers, the extra risk of ending up in the needy state during a recession epsido can be quite costly to the impatient agents, who in anticipation have lower expected welfare. However, comparing the results from table 14 to the welfare consequences in table 13, we see that the large welfare costs of eliminating the stabilizers were not due to business cycles. From the narrow business-cycle perspective of this paper, the stabilizers play a large role on how costly business cycles are. But, overall, these are not very costly to start with, so the benefit is small. Rather, the large welfare benefits of the stabilizers are due to redistribution and social insurance, not business-cycle fluctuations.

7 Conclusions and future work

Milton Friedman (1948) famously railed against the use of discretionary policy to stabilize the business cycle. He defended the power instead of fiscal automatic stabilizers as a preferred tool for countercyclical policy. More recently, Solow (2005) strongly argued that policy and research should focus more on automatic stabilizers as a route through which fiscal policy could and should affect the business cycle.

We constructed a business-cycle model with many of the stabilizers and calibrated it to replicate the U.S. data. The model has some interesting features in its own right. First, it nests both the standard incomplete markets model, as well as the standard new-Keynesian business cycle model. Second, it matches the first and second moments of U.S. business cycles, as well as the broad features of the U.S. wealth and income distributions. Third, solving it requires using new methods that may be useful for other models that combine nominal rigidities and incomplete markets.

We found that lowering taxes on sales, property, and corporate and personal income, or reducing the progressivity of the personal income tax, did not have a significant impact on the volatility of the business cycle. Moreover, lowering these taxes raised average output. At the same time, higher transfers to the unemployed and poor were quite effective at lowering the volatility of aggregate output.

In terms of the channels of stabilization, we found that the traditional disposable-income channel used to support automatic stabilizers is quantitatively weak. Considerably more
important was the role of precautionary savings and social insurance. Moreover, both because of the role of precautionary savings and because of the changes in government purchases to pay for public debt, the stabilizers can at the same time stabilize aggregate consumption, while destabilizing aggregate output.

Overall, we found that reducing the scope of all the stabilizers would have had little impact on the volatility of the U.S. business cycle in the last decades. This depends on monetary policy having responded aggressively to inflation and being close to optimal. When monetary policy is far from optimal, the automatic stabilizers play a useful role in reducing the magnitude of the business cycle, and during the recent episode with the zero lower bound, they may have significantly reduced the depth of the recession. Nearly all of the welfare impact of reducing the stabilizers comes from the social insurance they provide, and not from their impact on the business cycle.

Aside from monetary policy, labor market institutions and policies also likely matter for our results. In our model, we assume that transitions across employment states are exogenous, but this ignores the disincentive effects of transfers to those without a job on engaging in costly job search (e.g. Young, 2004) or in accepting lower wage offers (e.g. Hagedorn et al., 2013). We have explored this in a version of the model where the probability of finding a job depended on search effort, which in turn lowered leisure of unemployed households. As expected, stabilizers were even less effective, and more consistently destabilizing, as the increase in transfers to the unemployed during a recession lowers their search effort and prolongs the recession.23 Finally, Ravn and Sterk (2013) highlight an alternative interaction coming from labor demand, as unemployment risk leads to to precautionary savings, which lowers aggregate demand, reduces hiring, and so causes more unemployment risk. Like us, they find that aggressive monetary is very effective at dealing with this channel. Considering these and other interactions between the stabilizers and labor supply and demand seems a fruitful area for more research.

Another area for future work is the optimal design of stabilizers. Before doing so, we had to understand the positive predictions of the model regarding the stabilizers, a task that occupies this whole paper. Future work can take up the challenge of optimal policy design. Being able to do this work in a quantitative model like the one in this paper will have to overcome some challenging computational hurdles.

Finally, each of the automatic stabilizers that we considered is more complex than our description and distorts behavior in more ways than the ones we modeled. Here we have

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23 All of these experiments are available from the authors, or are in the working paper version of this paper.
sought to incorporate the most important channels through which the stabilizers could work while omitting other features of the economy in order to keep the model tractable. To obtain sharper quantitative estimate of the role of the stabilizers, it would be desirable to include the findings from the rich micro literatures that study each of these government programs in isolation. Perhaps the main point of this paper is that to assess automatic stabilizers requires having a fully articulated business-cycle model, so that we can move beyond the disposable-income channel, and consider other channels as well as quantify their relevance. Our hope is that as computational constraints diminish, we can keep this macroeconomic approach of solving for general equilibrium, while being able to consider the richness of the micro data.
References


Appendix

A From the NIPA tables to table 1

For each entry in table 1, we construct a sum of one or more entries in the NIPA tables, divide by nominal GDP, and average over 1988 to 2007. Here we describe the components of each entry in table 1.

A.1 Revenues

- **Personal income taxes** are the sum of federal and state income taxes (table 3.4) plus contributions for government social insurance less contributions to retirement programs (NIPA table 3.6, line 1 minus lines 4, 12, 13, 22, and 29).

- **Corporate income taxes** are from line 5 of table 3.1.

- **Property taxes** are the sum of business property taxes (table 3.5) and individual property taxes (table 3.4).

- **Sales and excise taxes** are state sales taxes (table 3.5) plus federal excise taxes (table 3.5).

- **Public deficit** is the residual between the two columns of the table.

- **Customs taxes** are from table 3.5, line 11.

- **Licenses, fines, fees** are the residual between current tax receipts from table 3.1 and the other revenue listed in our table.

- **Payroll taxes** are contributions to retirement programs (table 3.6, lines 4, 12, 13, 22, and 29).

A.2 Outlays

- **Unemployment benefits** are from table 3.12, line 7.

- **Safety net programs** are the sum of the listed sub-components from table 3.12, where “security income to the disabled” is the sum of lines 23, 29 and 36 and “Others” is the sum of lines 37 - 39.
• **Government purchases** are current consumption expenditure from table 3.1.

• **Net interest income** is the difference between interest expense and interest and asset income both from table 3.1.

• **Health benefits (non-retirement)** are spending on Medicaid (table 3.12, line 33). multiplied by the share of Medicaid spending that was spent on children, disabled, and non-elderly adults in 2007 plus other medical care (table 3.12, line 34).\(^{24}\)

• **Retirement-related transfers** are the share of Medicaid spent on the elderly plus Social Security, Medicare, pension benefit guarantees, and railroad retirement programs (all from table 3.12).

• **Other outlays** are the difference between total outlays in table 3.1 and those listed here.

**B Calibration of the idiosyncratic shock processes**

Each household at every date has a draw of \(s_t(i)\) determining the wage they receive if they are employed, and a draw of \(e_t(i)\) on their employment status. This section describes how we calibrate the distribution and dynamics of these two random variables.

**B.1 Skill shocks**

We use PSID data on wages to calibrate the skill process. To do this, we start with sample C from Heathcote et al. (2010a) and work with the log wages of household heads in years 1968 to 2002. Computational considerations limit us to three skill levels and we construct a grid by splitting the sample into three groups at the 33\(^{rd}\) and 67\(^{th}\) percentiles and then using the median wage in each group as the three grid points, which results in skill levels of 0.50, 0.92, and 1.64.\(^{25}\) Skills are proportional to the level (not log) of these wages. Computational considerations also lead us to choose a skill transition matrix with as few non-zero elements


\(^{25}\)The overall scale of skills is normalized so that average income in the economy is equal to one.
as possible. We impose the structure

\[
\begin{pmatrix}
1 - p & p & 0 \\
p & 1 - 2p & p \\
0 & p & 1 - p
\end{pmatrix},
\]

where \( p \) is a parameter that we calibrate as follows. From the PSID data, we compute the first, second and fourth auto-covariances of log wages. Let \( \Gamma_i \) be the \( i^{th} \) auto-covariance. We use the moments \( \Gamma_2/\Gamma_1 \) and \( \sqrt{\Gamma_4/\Gamma_2} \), each of which can be viewed as an estimate of the autoregressive parameter if the log wages follow an AR(1) process.\(^{26}\) The empirical moments are 0.9356 and 0.9496, respectively. To map these moments into a value of \( p \), we minimize the equally-weighted sum of squared deviations between these empirical moments and those implied by the three-state Markov chain. As our time period is one quarter, while the PSID data are annual, we use \( \Gamma_8/\Gamma_1 \) and \( \sqrt{\Gamma_{16}/\Gamma_8} \) from the model. This procedure results in a value of \( p \) of 0.015.

**B.2 Employment shocks**

**Steady state** In addition to differences in skill levels, households differ in their employment status. A household can be (1) employed (E), (2) unemployed (U) or (3) needy (N). To construct a steady state transition matrix between these three states we need six moments. First, it is reasonable to assume that a household does not transit directly from employed to long-term unemployed or from long-term unemployed back to unemployed. Those two elements of the transition matrix are therefore set to zero.

The distribution of households across states gives us two more moments. As the focus of our work is on the level and fluctuation in the number of individuals receiving different types of transfers, we define unemployed as individuals who are receiving unemployment benefits and needy as those receiving food stamps.

In the U.S., the Supplemental Nutritional Assistance Program is the largest non-health, non-retirement social safety net program. SNAP assists low-income households in being able to purchase a minimally adequate low-cost diet. Recipients of these benefits are generally not working.\(^{27}\) One virtue of using SNAP participation as a proxy for long-term unemploy-

\(^{26}\)The ratio \( \Gamma_1/\Gamma_0 \) is not used as this ratio is heavily influenced by measurement error, which leads to an underestimate of the persistence of wages. The moments that we use are also used by Heathcote et al. (2010b) to estimate the persistence of the wage process.

\(^{27}\)In 2009, 71% of SNAP recipient households had no earned income and only 17% had elderly individuals.
ment is that it avoids the subtle distinction between unemployment and non-participation in the Current Population Survey while still focussing on those individuals who likely have poor labor market prospects. If we instead used time since last employment to identify those in long-term unemployment, we would include a number of individuals with decent opportunities to work if they chose to do so such as individuals who have retired or who choose to work in the home. Between 1971, when the data begin, and 2011, the average insured unemployment rate was 2.9%. Between 1974, when the SNAP program was fully implemented nationwide, and 2011, the average ratio of SNAP participation to the insured labor force was 8.7%. We refer to this as the SNAP ratio.

Our final two moments speak to the flows across labor market states. We calibrate the flow into unemployment using the ratio of initial claims for unemployment insurance to the stock of employed persons covered by unemployment insurance. Between 1971 and 2011, the average value of this ratio was 5.16%. Many spells of unemployment insurance receipt are short and such spells are an important component of the data on flows. In our model, the minimum unemployment spell length is one quarter so we take care to account for the short spells in the data as part of our calibration strategy. We imagine that when a worker separates from their job, they immediately join the pool of job seekers and can immediately regain employment without an intervening (quarter-long) period of unemployment. To identify the probability of immediate reemployment, we assume it is the same as the job finding probability of other unemployed workers. In addition, we calibrate the probability of transitioning from long-term unemployment to employment based on the finding of Mabli et al. (2011) that 3% of SNAP participants leave the program each month.

Our procedure is as follows: we use the moments above to create a target transition matrix across employment states that our model should generate. This transition matrix has the form:

\[
\begin{pmatrix}
E & U & N \\
1 - s_1(1 - f_2) & s_1(1 - f_2) & 0 \\
\frac{f_2}{1 - f_2} & (1 - f_2)(1 - s_2) & (1 - f_2)s_2 \\
\frac{f_3}{1 - f_3} & 0 & 1 - f_3
\end{pmatrix}
\]

where element \((i, j)\) is the probability of moving from state \(i\) to state \(j\). There are four

---

(Leftin et al., 2010).

28 The insured unemployment rate is the ratio of the number of individuals receiving unemployment insurance benefits to the number of employed workers covered by unemployment insurance.

29 This ratio is calculated as the number of SNAP participants divided by the sum of the number of workers covered by unemployment insurance and the number of individuals receiving UI benefits.

30 In a typical quarter, the number of people who file an initial claim for UI is greater than the stock of recipients at a point in time.
parameters here $s_1, s_2, f_2, f_3$, which we set as follows: $f_3 = 0.0873$, equivalent to 3% per month; $s_1 = 0.0516$ is the ratio of initial claims to covered employment; $f_2 = 0.540$ and $s_2 = 0.577$ are chosen so the invariant distribution of the Markov chain matches the average shares of the population in each state.

**Business-cycle dynamics of employment risk**  
An important component of our model is the evolution of labor market conditions over the business cycle. One effect of the fluctuations in labor market conditions is to alter the number of households receiving different types of benefits over the cycle. A second effect is to alter the amount of risk that households face, which has consequences for the consumption and work decisions.

As we analyze the aggregate dynamics of the model with a linear approximation around the stationary equilibrium, it is sufficient to specify how the labor market risk evolves in the neighborhood of the stationary equilibrium. Let $\Pi_t$ be the matrix of transition probabilities between employment states at date $t$ and $t + 1$. We impose the following structure on the evolution of $\Pi_t$

$$\Pi_t = \Pi^0 + \Pi^1 \left[ \chi_1 \log z_t - \chi_2 \varepsilon_t - (1 - \chi_1 - \chi_2) \mu_t \right],$$

where $\Pi^0$ and $\Pi^1$ are constant $3 \times 3$ matrices. $\Pi^0$ is the matrix of transition probabilities between employment states in steady state. The term in brackets is a composite of the technology and labor market shocks and the parameter $\chi_1$ and $\chi_2$ control how much the labor market is driven by the three aggregate shocks. We set $\chi_1$ and $\chi_2$ so that the technology shocks account for 50% of the variance of the unemployment rate in keeping with the view that they drive 50% of the variance of output and the other two shocks each explain 25% of the variance of unemployment.

What remains is to specify the matrix $\Pi^1$.\(^{31}\) We use a $\Pi^1$ that has two non-zero, off-diagonal elements that allow the probability of losing employment to be counter-cyclical and allow the probability of moving from long-term unemployment to employment to be procyclical. We limit ourselves to these two parameters so as to economize on the number of parameters that must be calibrated. We choose these two elements of $\Pi^1$ to match the standard deviations of the insured unemployment rate and the SNAP ratio defined above.

The standard deviation of the insured unemployment rate is 0.00937 and the standard deviation of the SNAP ratio is 0.0205. These procedures leave us with the following:

\(^{31}\)The rows of $\Pi^1$ must sum to zero so that the rows of $\Pi_t$ always sum to one.
\[ \Pi^0 = \begin{pmatrix} 0.9694 & 0.0306 & 0 \\ 0.5398 & 0.1948 & 0.2654 \\ 0.0873 & 0 & 0.9127 \end{pmatrix}, \quad \Pi^1 = \begin{pmatrix} 2.81 & -2.81 & 0 \\ 0 & 0 & 0 \\ 2.33 & 0 & -2.33 \end{pmatrix}, \]

where the \((i, j)\) element of the \(\Pi\) matrices refers to the transition probability from state \(i\) to state \(j\) and the states are ordered as employed, unemployed, long-term unemployed. In addition, we have \(\chi_1 = 0.64\), and \(\chi_2 = 0.32\).

**C Decision problems and model equations**

In this section of the appendix, we derive the optimality conditions that we use to compute the equilibrium of the model.

**C.1 Patient household’s problem**

The patient household chooses \(\{c_t, n_t\}\) to maximize expression (1) subject to equations (2) and (3). Define \(\tilde{b}_t = b_t/p_t\) and \(\pi_t = p_t/p_{t-1}\) and note that \(\hat{p}_t/p_t = 1 + \tau^c\). Then we can rewrite the constraints as:

\[
(1 + \tau^c)c_t + \tilde{b}_{t+1}\pi_{t+1} - \tilde{b}_t = x_t - \bar{\tau}^x(x_t) + T_t^e
\]

\[
x_t = I_{t-1}\tilde{b}_t + w_t\bar{s}n_t + d_t.
\]

Setting up the Lagrangian, with \(m_t^1\) and \(m_t^2\) as the Lagrange multipliers on constraints (29) and (30), respectively, the optimality conditions are:

\[
\beta^t c_t^{-1} = m_t^1 (1 + \tau^c)
\]

\[
m_t^1 \pi_{t+1} = E_t \left[ m_{t+1}^1 + I_{t+1}m_{t+1}^2 \right]
\]

\[
m_t^2 = m_t^1 (1 - \tau^x(x_t))
\]

\[
\beta^t \psi_1 n_t^{\psi_2} = m_t^2 w_t\bar{s},
\]

These can be rearranged to give:

\[
\psi_1 n_t^{\psi_2} = \left( \frac{1}{c_t} \right) \left( \frac{1 - \tau^x(x_t)}{1 + \tau^c} \right) w_t\bar{s}, \quad (31)
\]

\[
\frac{1}{c_t} = \beta E_t \left\{ \frac{1 + I_t (1 - \tau^x(x_{t+1}))}{c_{t+1}\pi_{t+1}} \right\}, \quad (32)
\]
which are the patient household’s labor-supply and Euler conditions. Finally, notice that
the patient household’s stochastic discount factor is:
\[ \lambda_{t,s} = \frac{m_{t+s}^2}{m_t^2} = \frac{\beta^s c_{t+s}^{-1} (1 - \tau^x(x_{t+s}))}{c_t^{-1} (1 - \tau^x(x_t))}. \] (33)

C.2 Impatient households’ problem

The idiosyncratic state of a household is its real bond holdings \( \tilde{b} \), its employment status \( e \) and its skill level \( s \). Let \( S \) be the collection of aggregate state variables. Then the problem
of a household with real assets \( \tilde{b} \) and labor market states \( e \) and \( s \) can be written as

\[
V(\tilde{b}, e, s, S) = \max_{c, n} \left\{ \log(c) - \psi_1 n^{1+\psi_2} + \beta \mathbb{E} V(\tilde{b}', e', s', S') \right\}
\]

subject to

\[
(1 + \tau^c)c + \tilde{b}' \pi' - \tilde{b} = x - \tau^x(x) + T^s(j)
\]

\[
x = I(S_{-1})\tilde{b} + s(j) w(S)n + T^u(j),
\]

\[
n = 0 \quad \text{if } e \neq 2
\]

where \( i(S_{-1}) \) refers to the interest rate determined in the previous period. Here the expectation
operator is over aggregate and idiosyncratic shocks. From this problem, one can derive
an Euler equation and a labor supply condition that are analogous to those for the patient
household’s problem. One difference, however, is that in these analogous expressions the
expectation operator reflects an expectation over idiosyncratic uncertainty as well as over
aggregate uncertainty.

C.3 Intermediate Goods’ Firm

A firm that sets its price at date \( t \) chooses \( p_t^s, \{y_s(j), k_s(j), l_s(j)\}_{s=1}^{\infty} \) to solve

\[
\max E_t \sum_{s=t}^{\infty} \lambda_{t,s} (1 - \theta)^{s-t} \left\{ (1 - \tau^k) \left[ \frac{p_t^s}{p_s} y_s(j) - w_j l_s(j) - (\nu r_s + \delta) k_s(j) - \xi \right] - (1 - v) r_s k_s(j) \right\},
\]
subject to

\[ y_s(j) = \left( \frac{p_t^s}{p_s} \right)^{\mu/(1-\mu)} \ y_s \]

\[ y_s(j) = a_s k_s(j)^{\alpha} l(j)^{1-\alpha}. \]

where the first constraint is the demand for the firm’s good and the second its production function. By defining \( \hat{r}_t \equiv \left( 1 - \nu \tau^k \right) / (1 - \tau^k) r_t \), we can we can rewrite the objective function as if all capital costs were deductible, but the cost of capital were higher (\( \hat{r}_t > r_t \) if \( \nu < 1 \)). Dropping the constant \( 1 - \tau^K \) and substituting in the demand curve gives the modified problem:

\[
\max_{p_t^s, \{k_s(j),l_s(j)\}} \sum_{s=t}^{\infty} \left[ \left( \frac{p_t^s}{p_s} \right)^{1/(1-\mu)} y_s - w_s l_s(j) - (\hat{r}_s + \delta) k_s(j) - \xi \right] \lambda_{t,s} (1 - \theta)^{s-t}
\]

subject to

\[ \left( \frac{p_t^s}{p_s} \right)^{\mu/(1-\mu)} y_s = a_s k_s(j)^{\alpha} l(j)^{1-\alpha}. \]

The first order conditions with respect to \( k_s(j) \) and \( l_s(j) \) are:

\[ (\hat{r}_s + \delta) = M_s \alpha a_s k_s(j)^{\alpha-1} l_s(j)^{1-\alpha}. \] \hfill (34)

\[ w_s = M_s (1 - \alpha) a_s k_s(j)^{\alpha} l_s(j)^{-\alpha}, \] \hfill (35)

where \( M_s \) is the Lagrange multiplier on the production function constraint at date \( s \), which is real marginal cost at date \( s \).

We can derive several useful features of the solution from these two optimality conditions. First, taking their ratio:

\[ \frac{w_s}{\hat{r}_s + \delta} = \frac{1 - \alpha}{\alpha} \frac{k_s(j)}{l_s(j)}, \]

so that all firms have the same capital-labor ratio and, by market clearing, \( k_s(j) / l_s(j) = k_s / l_s \) for all firms.

Second, these optimality conditions allow us already to derive the expression for dividends
as a function of factor prices. Total factor payments are

\begin{align}
(\hat{r}_s + \delta) k_s &= M_s \alpha a_s k_s^{1-\alpha}, \\
w_s l_s &= M_s (1 - \alpha) a_s k_s^{1-\alpha}.
\end{align}

The aggregate after-tax dividend of the intermediate goods firms is then

\[
\int_0^1 d_t(j) dj = (1 - \tau^k) \int_0^1 \left[ \frac{p_t(j)}{p_t} y_t(j) - w_t(j) l_t(j) - (\hat{r}_t + \delta) k_t(j) - \xi \right] dj
\]

and by market clearing this becomes

\[
\int_0^1 d_t(j) dj = (1 - \tau^k) \left[ y_t - M_t a_t k_t^{1-\alpha} - \xi \right].
\]

Similarly, total profits are

\[
\int_0^1 \left[ \frac{p_t(j)}{p_t} y_t(j) - w_t l_t(j) - (r_t + \delta) k_t(j) - \xi \right] dj \\
= \int_0^1 \left[ \frac{p_t(j)}{p_t} y_t(j) - w_t l_t(j) - (\hat{r}_t + \delta + r_t - \hat{r}_t) k_t(j) - \xi \right] dj \\
= y_t - M_t a_t k_t^{\alpha(1-\alpha)} - \xi + \tau^k \frac{1 - \nu}{1 - \nu \tau^k} \hat{r}_t k_t.
\]

And revenue from the corporate income tax is the difference between (39) and (38).

Finally, we turn to the optimality condition with respect to \( p_t^* \):

\[
\mathbb{E}_t \sum_{s=t}^{\infty} \lambda_{t,s} (1 - \theta)^{s-t} \left[ \frac{1}{1 - \mu} \left( \frac{p_t^*}{p_s} \right)^{1/(1-\mu)-1} \frac{y_s}{p_s} - M_s \frac{\mu}{1 - \mu} \left( \frac{p_t^*}{p_s} \right)^{\mu/(1-\mu)-1} \frac{y_s}{p_s} \right] = 0,
\]

which we can rewrite as

\[
\mathbb{E}_t \sum_{s=t}^{\infty} \frac{1}{1 - \mu} \left( \frac{p_t^*}{p_s} \right)^{1/(1-\mu)-1} \frac{y_s}{p_s} \lambda_{t,s} (1 - \theta)^{s-t} = \mathbb{E}_t \sum_{s=t}^{\infty} \lambda_{t,s} (1 - \theta)^{s-t} M_s \frac{\mu}{1 - \mu} \left( \frac{p_t^*}{p_s} \right)^{\mu/(1-\mu)-1} \frac{y_s}{p_s}
\]

\[
\frac{p_t^*}{p_t} = \frac{p_t \mathbb{E}_t \sum_{s=t}^{\infty} \lambda_{t,s} (1 - \theta)^{s-t} M_s \mu_t \left( \frac{p_s}{p_t} \right)^{\mu/(1-\mu)-1} \frac{y_s}{p_s}}{p_t \mathbb{E}_t \sum_{s=t}^{\infty} \left( \frac{p_s}{p_t} \right)^{\mu/(1-\mu)-1} \frac{y_s}{p_s} \lambda_{t,s} (1 - \theta)^{s-t}} \equiv \frac{\bar{p}_t^A}{\bar{p}_t^B}.
\]
This equation gives the solution for $p_t^*$. It is useful to write $\bar{p}_t^A$ and $\bar{p}_t^B$ recursively. To that end,

$$\bar{p}_t^A = p_t \mathbb{E}_t \sum_{s=t}^{\infty} \lambda_{t,s} (1 - \theta)^{s-t} M_s \mu_t \left( \frac{p_t}{p_s} \right)^{\mu/(1-\mu) - 1} \frac{y_s}{p_s}$$

$$= M_t \mu_t y_t +$$

$$\mathbb{E}_t p_{t+1} \pi_{t+1}^{-1} \mathbb{E}_t \lambda_{t,t+1} (1 - \theta) \left( \frac{p_t}{p_{t+1}} \right)^{\mu/(1-\mu) - 1}$$

$$\times \sum_{s=t+1}^{\infty} \lambda_{t+1,s} (1 - \theta)^{s-t-1} M_s \mu \left( \frac{p_{t+1}}{p_s} \right)^{\mu/(1-\mu) - 1} \frac{y_s}{p_s}$$

$$= M_t \mu_t y_t + \mathbb{E}_t \left[ \lambda_{t,t+1} (1 - \theta) \pi_{t+1}^{-\mu/(1-\mu)} \bar{p}_t^A \right], \quad (41)$$

where $\pi_{t+1} \equiv p_{t+1}/p_t$. Similar logic for $\bar{p}_t^B$ yields

$$\bar{p}_t^B = y_t + \mathbb{E}_t \left[ \lambda_{t,t+1} (1 - \theta) \pi_{t+1}^{-\mu/(1-\mu)} \bar{p}_{t+1}^B \right]. \quad (42)$$

Next, comes the relationship between $p_t^*$ and inflation. The price index is

$$p_t = \left( \int_0^1 p_t(j)^{1/(1-\mu)} \, dj \right)^{1-\mu}$$

and with Calvo pricing we have

$$p_t = (1 - \theta) \int_0^1 (p_{t-1}(j))^{1/(1-\mu)} \, dj + \theta (p_t^*)^{1/(1-\mu)} \right)^{1-\mu}$$

$$= (1 - \theta) p_t^{1/(1-\mu)} + \theta (p_t^*)^{1/(1-\mu)} \right)^{1-\mu}.$$

Therefore

$$\pi_t = \left( \frac{1 - \theta}{1 - \theta \left( \frac{p_t^*}{p_t} \right)^{1/(1-\mu)}} \right)^{1-\mu} \quad (43)$$

Finally, note that because the capital-labor ratio is constant across firms, the production of variety $j$ follows:

$$y_t(j) = a_t \left( \frac{k_t}{l_t} \right)^{\alpha} l_t(j).$$
The demand for variety \( j \) can be written in terms of the relative price to arrive at

\[
\left( \frac{p_t(j)}{p_t} \right)^{\mu/(1-\mu)} y_t = a_t \left( \frac{k_t}{\ell_t} \right)^\alpha \ell_t(j).
\]

Integrating both sides yields

\[
\int_0^1 \left( \frac{p_t(j)}{p_t} \right)^{\mu/(1-\mu)} \, dj y_t = a_t \left( \frac{k_t}{\ell_t} \right)^\alpha \int_0^1 \ell_t(j) \, dj.
\]

By market clearing we have then that:

\[
S_t y_t = a_t k_t^\alpha \ell_t^{1-\alpha},
\]

where

\[
S_t = \int_0^1 \left( \frac{p_t(j)}{p_t} \right)^{\mu/(1-\mu)} \, dj.
\]

\( S_t \) reflects the efficiency loss due to price dispersion and it evolves according to

\[
S_t = (1 - \theta) S_{t-1} \mu/(1-\mu) + \theta \left( \frac{p_t^*}{p_t} \right)^{\mu/(1-\mu)}.
\]

Throughout this subsection, we have dropped most of the \( t \) subscripts on \( \mu_t \). When the equations in this subsection are linearized around the zero-inflation steady state, the markup shock only enters equation (41).

### C.4 Capital goods firm

The capital goods firms chooses a sequence \( \{k_{t+1}, k_{t+2}, \cdots \} \) to maximize

\[
\mathbb{E}_t \sum_{s=t}^{\infty} \lambda_{t,s} (1 + \tau^F)^{-(s-t+1)} \left[ r_s k_s - k_{s+1} + k_s - \frac{\zeta}{2} \left( \frac{k_{s+1} - k_s}{k_s} \right)^2 k_s \right].
\]

The discounting by \( 1/(1 + \tau^F) \) comes from the property tax since:

\[
v_t = \frac{1}{1 + \tau^F} d_t^k + \frac{1}{1 + \tau^F} \mathbb{E}_t [\lambda_{t,t+1} v_{t+1}].
\]
This problem leads to the first-order condition

\[
1 + \zeta \left( \frac{k_{t+1} - k_t}{k_t} \right) = \mathbb{E}_t \left\{ \frac{\lambda_{t,t+1}}{1 + \tau^p} \left[ r_{t+1} + 1 - \frac{\zeta}{2} \left( \frac{k_{t+2} - k_{t+1}}{k_{t+1}} \right)^2 + \zeta \left( \frac{k_{t+2} - k_{t+1}}{k_{t+1}} \right) \frac{k_{t+2}}{k_{t+1}} \right] \right\}.
\]

(47)

This expression can be transformed into one that only includes variables dated \( t \) and \( t + 1 \) by writing it in terms of \( \hat{k}_t = k_{t+1} \) and introducing \( \hat{k}^{\text{lag}}_t = \hat{k}_{t-1} \). Dividends paid by the capital goods firm are the term in brackets in the objective function less \( \tau^p \) times the value of the firm, which follows equation (46).

D Proofs for propositions

Proof of proposition 1. Before turning to the full proof, we highlight the intuition for the result. With flexible prices, there is an aggregate Cobb-Douglas production function, so if the capital stock and employment are fixed, then the proposition will be true as long as the labor supply is fixed. Equating the marginal rate of substitution between consumption and leisure for households to their after-tax wage gives the standard labor supply condition:

\[
n_t(i) = \left( \frac{(1 - \tau^x) s_t(i) w_t}{\psi_1 c_t(i)(1 + \tau^c)} \right)^{1/\psi_2}
\]

Perfect insurance implies that consumption is equated across households. But then, our balanced-growth preferences and technologies imply that \( c_t/w_t \) is fixed over time, so the condition above, once aggregated over all households, gives a constant labor supply.

The full proof goes as follows. Under complete markets, the households will fully insure idiosyncratic risks. Therefore, we treat them as a large family that pools risks among its members. In determining the family’s tax bracket, we assume the tax collector applies the tax rate corresponding to the average income of its members.

The large family maximizes

\[
\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[ \ln(c_t) - \psi_1 \frac{n_t^{1+\psi_2}}{1 + \psi_2} + \int_0^\nu \ln c_t(i) - \psi_1 \frac{n_t(i)^{1+\psi_2}}{1 + \psi_2} di \right]
\]

65
subject to

\[ \hat{p}_t \left[ \int_0^\nu c_t(i)di + c_t \right] + b_{t+1} - b_t = p_t [x_t - \bar{\tau}(x_t)] + T_t, \]

where \( T_t \) is net non-taxable transfers to the household and

\[ x_t = (I_{t-1}/p_t)b_t + w_t\bar{s}n_t + d_t + \int_0^\nu s_t(i)n_t(i) + T_t(i)di. \]

The household also faces the constraint \( n_t(i) = 0 \) if \( e_t(i) \neq 2 \). Let \( m^1_t \) be the Lagrange multiplier on the former constraint and \( m^2_t \) be the Lagrange multiplier on the latter. Then the first order conditions of this problem are

\[ \frac{\beta^t}{c_t} = \hat{p}_t m^1_t, \]

\[ \frac{\beta^t}{c_t(i)} = \hat{p}_t m^1_t, \]

\[ m^1_t = \mathbb{E}_t \{ m^1_{t+1} + m^2_{t+1}(I_t/p_t) \} \]

\[ m^1_t p_t [1 - \tau^x(x_t)] = m^2_t \]

\[ \beta^t \psi_1 n^\psi_2 = m^2_t w_t\bar{s} \]

\[ \beta^t \psi_1 n_t(i)\psi_2 = m^2_t w_t s_t(i) \]

These first order conditions can be rearranged to obtain

\[ c_t(i) = c_t, \]

\[ \frac{1}{c_t} = \beta \mathbb{E}_t \left\{ \frac{1 + I_t [1 - \tau^x(x_{t+1})]}{c_{t+1}\bar{\pi}_{t+1}} \right\}, \]

and aggregate labor input satisfies

\[ \bar{s}n_t + \int_0^\nu s_t(i)n_t(i)di = \left\{ \frac{1}{\psi_1 c_t} \frac{1 - \tau^x(x_t)}{1 + \tau^c} w_t \right\}^{1/\psi_2} \left[ \bar{s}^{1+1/\psi_2} + E_t \int_0^\nu (s_t(i))^{1+1/\psi_2} di \right], \]

where \( E_t \) is defined as the mass of impatient households who are employed. In this final step we should only integrate over those households that are not at a corner solution, but this is trivial as the marginal disutility of labor goes to zero as \( n_t(i) \) goes to zero so all households with positive wages are employed and it is only those who exogenously lack employment
opportunities who will set \( n_t(i) = 0 \).

Proceeding similarly for the representative agent decision problem stated in the proposition and defining aggregate labor input in that case to be \((1 + E_t)s_t n_t\), one reaches the conclusion that the two models will deliver the same Euler equation and condition for aggregate labor supply. Therefore, the two models will generate the same aggregate dynamics.  \( \square \)

**Proof of proposition 2.** Under assumption 1, we can use the representative agent formulation from proposition 1. The labor supply condition for this problem is

\[
\dot{n}_t = \left( \frac{(1 - \tau^x) w_t s_t}{c_t (1 + \tau^c) \psi_1} \right)^{1/\psi_2},
\]

where \( \tau^x \) is the (constant) marginal tax rate. Under the conditions of assumption 2, the aggregate resource constraint is: \( c_t + g_t = y_t \). But, since there is a constant ratio of \( g_t \) to \( y_t \), the resource constraint implies that \( c_t/y_t \) is constant and equal to \( 1 - \bar{g}/\bar{y} \). Moreover, with flexible prices, we can write \( w_t = \frac{(1 - \alpha) y_t}{\mu L_t} \), where \( L_t \) is aggregate labor input. Using these two results to substitute out \( c_t \) and \( w_t \) we obtain

\[
n_t = \left[ \frac{(1 - \tau^x)(1 - \alpha) y_t}{(1 - \bar{g}/\bar{y}) y_t (1 + \tau^c) \psi_1 \mu n_t (1 + E)} \right]^{1/\psi_2},
\]

where we have used the fact that the aggregate labor input is \( n_t s_t (1 + E) \), where employment is constant by assumption. Using this expression, we can solve for \( n_t \) as

\[
n_t^{1 + 1/\psi_2} = \left[ \frac{(1 - \tau^x)(1 - \alpha)}{(1 - \bar{g}/\bar{y}) (1 + \tau^c) \psi_1 \mu (1 + E)} \right]^{1/\psi_2}.
\]

Because the right-hand-side does not depend on time, it follows that \( n_t \) is constant over time.

Next, recall that capital is fixed and prices are flexible, so aggregate output is

\[
y_t = a_t K^\alpha [(1 + E) s n]^{1 - \alpha},
\]

where \( K \) and \( n \) are the constant inputs of capital and hours, \( 1 + E \) is total employment and \( s \) is the skill level of the representative agent, which is also constant over time by the fact that the labor market risk is unchanging over time so the composition of the pool of workers is stable. It follows from this equation that the variance of log output is equal to the variance of log productivity, \( a_t \).

That \( S = 0 \) follows from the fact that the productivity process is exogenous and therefore
not affected by the presence or absence of automatic stabilizers. Notice that $S = 0$ holds regardless of whether one uses output or consumption as the measure of activity as $c_t/y_t$ is constant. For hours, the ratio is not defined since there is no variation in hours worked.

### E Numerical solution algorithm

As the main text described, the key steps involved in solving the model are: (i) to discretize the cross-sectional distributions and decision rules, (ii) to solve for the stationary equilibrium, (iii) to collect all of the many equations defining an approximate equilibrium and linearizing them, and (iv) solve the system with a linear rational expectations solver. We elaborate on each of these steps next.

#### E.1 Discretizing the model

For each discrete type of impatient household characterized by a skill level and an employment status, we approximate the distribution of wealth by a histogram with 250 bins. We approximate the policy rules for savings and labor supply by two piece-wise linear splines with 100 knot points each. We deal with the borrowing constraint in the approximation of the policy functions by, following Reiter (2010), parameterizing the point at which the borrowing constraint is just binding, and then constructing a grid for higher levels of assets. As a result of these approximations, there are now 450 variables for employed workers, and 350 variables for non-employed workers (who do not choose labor supply).

#### E.2 Solving for the stationary equilibrium

Solving for the steady state of the model requires two steps: first, solving for the impatient household policy rules and distribution of wealth and second, solving for the aggregate variables including the assets and consumption of the representative patient household. These two steps are interrelated as the equilibrium interest rate depends on the patient household’s marginal tax rate, which depends on the patient household’s income and therefore wealth, which in turn depends on the level of wealth held by impatient households.

We use an iterative procedure to find the equilibrium income of the patient households. Given a guess of the patient household’s income and therefore marginal tax rate, we find the equilibrium interest rate from the patient household’s Euler equation and then the solution
of the intermediate goods firm’s problem to find the equilibrium wage. With these objects, we solve the impatient households’ problem to find their consumption and asset positions. With these in hand, we use standard techniques from the analysis of representative agent models to find the rest of the aggregate variables. Finally, we check our guess of the patient household’s income and iterate from here.

E.3 System of equations

Keeping track of the wealth distribution We track real assets at the beginning of the period using Reiter’s (2010) procedure to allocate impatient households to the discrete grid in a way that preserves total assets. As we have nominal bonds in the model, we account for the effect of inflation in the evolution of the household’s asset position. For each discrete type of household this provides 250 equations.

Solving for household decision rules We use the impatient household’s Euler equation and labor supply condition to solve for their decision rules by imposing that these equations hold with equality at the spline knot points. This provides 100 equations for non-employed households and 200 for employed households.

Aggregate equations In addition to those equations that relate to the solution of the impatient household’s problem and the distribution of wealth across households, we have equations that correspond to the patient household’s savings and labor supply decisions, as well as those that correspond to the firms’ problems. These equations are discussed in more detail in Appendix B. We use equations (29), (31), (32), (40), (41), (42), (43), (36), (37), (44), (45), (46), (47). We introduce an auxiliary variable that carries an extra lag of capital, $k_t^{\text{lag}} = k_{t-1}$. In addition, from the main text we have equations (22), (23), (24), (26), (25), (27), and exogenous AR(1) processes for $\epsilon_t$, $a_t$, and $\mu_t$. We use these equations to solve for $c_t$, $n_t$, $b_t$, $M_t$, $p_t^*/p_t$, $\bar{p}_t^A$, $\bar{p}_t^B$, $S_t$, $\pi_t$, $w_t$, $r_t$, $v_t$, $k_t$, $k_t^{\text{lag}}$, $d_t$, $B_t$, $T_t^e$, $g_t$, and $I_t$.

E.4 Linearization and solution

At this stage, we have a large system of non-linear equations that the discretized model must satisfy. We follow Reiter (2009, 2010) in linearizing this system around the stationary equilibrium using automatic differentiation and then solving the linearized system as a linear rational expectations model using the algorithm from Sims (2002).
F Numerical error analysis

Here we discuss the accuracy of our numerical calculations for the main results, in section 4. There are two sources of errors both of which commonly arise in related algorithms. First, there are errors in the decision rules of the impatient households between the points at which the household optimality conditions are imposed. These errors are present even in the stationary equilibrium. Away from the stationary equilibrium (the point around which we linearize) there are errors due to non-linear responses to aggregate states as is the case with other applications of perturbation methods.

To assess the accuracy of our solution, we calculate unit-free Euler equation errors.\(^{32}\) We calculate the Euler equation errors for the patient household as well as for impatient households. For impatient households we use a test grid over asset holdings that is finer than the grid on which we solve for household decision rules.\(^{33}\) For a given aggregate state of the economy, \(S_t\), the distribution of bond holdings, the capital stock, and exogenous variables are predetermined.

Pre-determined and exogenous: \(k_t, B_t, b_t, a_t, \varepsilon_t, \mu_t\), distribution of households.

We then use the computed solutions to determine

Approx. solutions: \(M_t, c_t, v_t, \bar{p}_t^A, \bar{p}_t^B\), impatient hhld. savings and labor supply rules.

We then use the non-linear, static relationships and market clearing conditions to determine the remaining variables. Table 15 lists the equations we impose and the variables that we solve for. In addition to those equations listed, the patient household budget constraint, equation (2), holds by Walras Law. The aggregate resource constraint is

\[
\begin{align*}
    k_{t+1} + c_t + \int_0^\nu c_t(i)di + g_t &= y_t + (1 - \delta)k_t - \xi - \frac{\zeta}{2} \left( \frac{k_{t+1} - k_t}{k_t} \right)^2 k_t
\end{align*}
\]  

Notice that all budget constraints and market-clearing conditions are forced to hold. From these calculations and a given set of aggregate shocks we can compute the next state of the economy, \(S_{t+1}\), and repeat these steps to find \(c_{t+1}\) and so on. To compute expectations, we use Gaussian quadrature over the three aggregate shocks using a grid that has 11 nodes in

\(^{32}\)See Judd (1992) for an explanation of this accuracy check and the interpretation of the errors in terms of bounded rationality.

\(^{33}\)Specifically, we use the same 250 point grid for \(b(i)\) as we use to approximate the distribution of wealth.
Table 15: Equations that hold exactly in error analysis. Due to the capital adjustment cost, there are two values of $k_{t+1}$ that solve the aggregate resource constraint, the relevant solution is the larger of the two.

Each dimension. For a given household (i.e. a patient household or an impatient household with particular idiosyncratic states) we can compute the level of consumption implied by the right-hand side of the Euler equation as

$$\hat{c}_t \equiv \left[ \beta \mathbb{E}_t \left\{ \frac{1 + I_t (1 - \tau^x(x_{t+1}))}{c_{t+1} \pi_{t+1}} \right\} \right]^{-1},$$

(49)

where the expectation is over aggregate and idiosyncratic shocks in the case of impatient households. The unit-free Euler equation error for a given type of household is then $\hat{c}_t/c_t - 1$, where $c_t$ is the level of consumption implied by the approximated decision rules.\(^{34}\) Here we have used the Euler equation for bond holdings, which is the relevant Euler equation for impatient households. For patient households we could alternatively use equation (47) to construct $\hat{c}$. We will refer to these two versions as the “bond” error and the “investment” error.

Using the steps above, we can compute the Euler equation error for each type of household. As a summary statistic, we integrate $\hat{c}$ across households using the distribution of wealth at the given state of the economy to compute aggregate consumption implied by the right-hand side of the Euler equation. We similarly can integrate the consumptions implied by the approximate policy rules to find aggregate consumption as implied by the left-hand

\(^{34}\)For impatient households we approximate their policy rules for savings as opposed to consumption so $c$ is computed from their budget constraint and depends on the approximate policy rule for labor supply and the market-clearing prices.
side of household Euler equations. We can then express an aggregate Euler equation error for all impatient households as \( \int \hat{c}(i)di / \int c(i)di - 1 \) and an aggregate Euler equation error for all households as \( \left[ \int \hat{c}(i)di + \hat{\hat{c}} \right] / \left[ \int c(i)di + c \right] - 1 \), where \( c \) is the consumption of the patient households and \( \hat{c} \) can be calculated from either (47) or (49). We choose to focus on these aggregate Euler equation errors as opposed to the disaggregated errors for each type of household because this is what is relevant to our results on aggregate dynamics. Nonetheless, the disaggregated Euler errors do not show large differences in magnitude across households.

We can also assess the errors in household labor supply decision rules. In the course of the steps listed above we have solved for everything on the right-hand side of equation (31) and the analogous equations for impatient households. Specifically, we use the approximate solutions to find \( c \) and the value of \( x \) that follows from plugging the approximate policy rule for \( n \) into equation (3). We can then solve for the implied value of \( n \), call it \( \hat{n} \), from the right-hand side of equation (31) and express the error in this equation as \( \hat{n}/n - 1 \). Again, we summarize these errors by integrating \( n \) and \( \hat{n} \) over the distribution of households.

The Euler equation and labor supply errors vary over the state space. We randomly draw points in the state space by simulating the model for 50,000 periods and we compute the errors every 1,000 simulated periods. We describe the distribution of errors across the 50 resulting points by reporting the largest absolute error and the mean absolute error in table 16.

<table>
<thead>
<tr>
<th></th>
<th>Euler equation errors (log base 10)</th>
<th></th>
<th>Labor supply errors (log base 10)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>patient</td>
<td>impatient</td>
<td>aggregate</td>
</tr>
<tr>
<td></td>
<td>investment bond</td>
<td>investment bond</td>
<td></td>
</tr>
<tr>
<td>largest</td>
<td>-2.15</td>
<td>-2.09</td>
<td>-2.05</td>
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<tr>
<td>mean</td>
<td>-3.10</td>
<td>-2.78</td>
<td>-2.78</td>
</tr>
</tbody>
</table>

Table 16: Largest and mean absolute errors across 50 randomly-drawn points in the state space.
G Methods for transition dynamics

In sections 5 and 6, we discuss perfect foresight transition experiments. Unlike above, we do not linearize the model equations, but instead compute the transition using the fully non-linear model equations.

Initial guess. We assume that the economy has returned to steady state after $T = 250$ periods and look for equilibrium values for endogenous variables between dates $t = 0$ to $T$. In this explanation of our methods we use variables without subscripts to represent sequences from 0 to $T$. Let $X$ denote a path for all endogenous aggregate variables from date 0 to date $T$. These variables include aggregate quantities and prices. Specifically, in Appendix E.3 under the heading “Aggregate equations” we list 20 endogenous aggregate variables and three exogenous variables. In addition to those we also include the exogenous path for the preference shock for the zero-lower-bound experiment. In addition, let $c_i^t \equiv \int_0^\nu c_t(i)di$ be the aggregate consumption of impatient households. Define $b_i^t$ similarly as the aggregate bond holding of the impatient households and $n_i^t \equiv \int_0^\nu s_t(i)n_t(i)di$ as the aggregate effective labor supply. Finally define $T_i^t \equiv \int x_t(i)di$ as the aggregate income tax payment of the impatient households. $X$ contains the sequences $\{c_i^t, n_i^t, b_i^t, T_i^t\}_{t=0}^T$. Importantly $X$ does not include the distribution of wealth or the household decision rules. So in total, $X$ represents time paths for 28 variables four of which are exogenous.

We require an initial guess $X^0$. We start with a scaled down version of the exogenous variables so that they differ from their steady state values by a small amount. For this starting point, the steady state values for quantities and prices are good initial guesses. After computing an equilibrium for this scaled down problem we then gradually scale up the exogenous variables to the full version of the transition experiment.

Solving the household’s problem. The impatient household’s decision problem depends on $X$ through the prices. For a given $X^i$ we solve the household’s problem using the endogenous gridpoint method (Carroll, 2006).

Simulating the population of households. We simulate the population of households in order to compute aggregate consumption and aggregate labor supply. We use a non-stochastic simulation method. We approximate the distribution of wealth with a histogram with 250 unequally-spaced wealth levels for each value of $(e, s)$ placing more bins at low asset levels. We then update the distribution of wealth according to the household savings
policies and the exogenous transitions across skill and employment states. When households choose levels of savings between the center of two bins, we allocate these households to the adjacent bins in a way that preserves total savings. See Young (2010) for a description of non-stochastic simulation in this manner.

Checking the equilibrium conditions. In Appendix E.3 under the heading “Aggregate equations” we list 20 equations. These equations depend on the distribution of wealth and the impatient household decision rules only through $c^I_t$, $n^I_t$, $b^I_t$, and $T^I_t$. So we can directly check whether these 20 equations hold at $X$. In addition, we need to verify that $c^I_t$, $n^I_t$, $b^I_t$, $T^I_t$ are part of an equilibrium. This requires solving for the household decision rules and simulating the population of impatient households. We do that using the methods described in the previous two paragraphs and check whether the aggregate behavior of impatient households that is implied matches the values listed in $X$.

Updating $X^i$. The difficult part of the solution method arises when our guess $X^i$ is not an equilibrium. In this case we need to find a new guess $X^{i+1}$ that moves us towards an equilibrium. To do this, we construct an auxiliary model by replacing the computational equilibrium conditions with additional analytical equilibrium conditions that approximate the behavior of the population of impatient households but are easier to analyze. Specifically we use the equations

$$(c^I_t)^{-\gamma} = \eta_1^I T^I_t (1 + \tau^I_t) (c^I_{t+1})^{-\gamma}$$

$$(c^I_t)^{-\gamma} w_t = \eta_2^I \psi$$

$$T^I_t = \eta_3^I (w_t n^I_t + I_{t-1} b^I_t / \pi_t + T^{u,I} U_t)$$

$$(1 + \tau^c) c^I_t + b^I_{t+1} = (1 + I_{t-1}) b_t / \pi_t + w_t n^I_t + T^{u,I} U_t + T^{n,I} N_t - \tau^I_t$$

where $\eta_1^I$, $\eta_2^I$, and $\eta_3^I$ are treated as parameters of the auxiliary model and $U_t$ and $N_t$ are masses of unemployed and needy households. $T^{u,I}$ and $T^{n,I}$ are parameters that determine the aggregate transfer payments as a function of the number of unemployed and needy. These are constant parameters as the distribution of skills is stationary. $U_t$ and $N_t$ evolve exogenously in line with the markov chain transition matrix $\Pi_t$.

35 For the zero lower bound experiment, we modify equation (27) such that the nominal interest rate is the maximum of the value implied by the Taylor rule and zero.

74
For a given $X^i$, we have computed $c^I$, $n^I$, $b^I$ and $T^I$ from the computational equilibrium conditions. We then calibrate $\eta^1$, $\eta^2$, $\eta^3$ from the above equations. We then solve for a new value of $X$ from the 20 analytical equilibrium conditions and this system of equations. This is a problem of solving for 26 unknowns at each date from 26 non-linear equations at each date.\footnote{There are 26 equations and endogenous variables as opposed to 24 because we include $N$ and $U$ and the associated equations.} We solve this system using the method described by Juillard (1996) for computing perfect foresight transition paths for non-linear models. This method is a variant of Newton’s method that exploits the sparsity of the Jacobian matrix. Call this solution $X^{i\prime}$. We then form $X^{i+1}$ by updating partially from $X^i$ towards $X^{i\prime}$.

In essence, we are computing an equilibrium as if there were a representative impatient household whose behavior were described by a standard Euler equation with the wedge $\eta^1$, a standard labor supply equation with the wedge $\eta^2$, and so on. We use this equilibrium under a representative impatient household to construct our next guess $X^{i+1}$. We iterate on these steps until the values $\eta^1$, $\eta^2$, and $\eta^3$ converge.