

# Absolute standardless calibration of photodetectors based on quantum two-photon fields

A. N. Penin and A. V. Sergienko

A new method of absolute calibration of photodetectors based on a nonclassical effect in nonlinear optics is discussed. The combined influence of a monochromatic flux of pump photons and quantum vacuum noise on a nonlinear crystal with no center of symmetry results in the appearance of rigorously correlated pairs of photons with continuous spectral distribution in the spontaneous parametric scattering of light (spontaneous parametric downconversion) process. The presence of an optical field in a broad spectral range created by the two-photon states (biphotons) makes it possible to develop a new method of measuring the spectral distribution of the absolute value of the quantum efficiency of photodetectors, both in photon counting and in analog regimes, that does not use any calibrated standard light sources (étalons). The feasibility of the method is demonstrated on different types of photomultiplier tube.

## Introduction

There are two basic problems in photometry—calibration of photodetector sensitivity and measurement of radiation brightness (i.e., spectral density and also intensity, energy, and power). These problems are usually solved by methods that use standard light sources that are blackbody radiation models with temperatures as great as 3000 K. Recently standard radiometers have appeared. The basic elements of standard radiometers are self-calibrated photodetectors and special detection systems with quantum efficiency near 100%.<sup>1</sup>

In the mid-1980's some important new methods of photodetector absolute calibration and energetic brightness (spectral density) measurement were proposed.<sup>2</sup> These methods use optical fields with nonclassical statistics.

If coherent radiation of frequency  $\omega_0$  is directed onto a nonlinear crystal without a center of symmetry, then each photon of this pump radiation can spontaneously disintegrate into two photons inside the sample with some probability. The frequencies of these photons

are both less than  $\omega_0$  and occupy a broad spectral range from the  $\omega_0$  down to the frequencies of the crystal lattice resonances. This effect was observed for the first time more than 20 years ago.<sup>3</sup> A detailed quantum-theoretical treatment of this phenomenon has been made<sup>4</sup> in which this effect was called the spontaneous parametric scattering of light (SPSL) process. This name, from our point of view, exactly reflects the physical nature of this phenomenon, describing the creation of optical radiation with continuous distribution in a broad spectral and angular range as a result of the interaction of pump radiation with the nonlinear crystal. The statistical properties of optical fields in the parametric processes of nonlinear optics were also investigated theoretically in detail.<sup>5</sup> In later publications<sup>6</sup> this phenomenon was called spontaneous parametric downconversion by analogy with the well-known parametric generation process.

The interest in the phenomenon of SPSL comes from its intrinsically quantum origin. Only a quantum-mechanical treatment can provide a complete and noncontradictory description of all parameters of this process.<sup>4-7</sup> The quantum nature of SPSL provides an opportunity not only for investigations of the quantum properties of electromagnetic fields but also for development of new methods for physics research and for the design of new devices with properties that exceed those of their classical analogs.

The SPSL process (in which the parametric coupling constant  $k \ll 1$ ) and the parametric conversion process ( $k = 1$ ) are the preferred processes now for the realization of nonclassical effects in quantum optics. The generation of sub-Poissonian photon flux has

---

When this research was done the authors were with the Department of Physics, Moscow University, Moscow 119899, USSR. A. V. Sergienko is now with the Department of Physics and Astronomy, University of Maryland, College Park, Maryland 20742.

Received 30 January 1990.

0003-6935/91/253582-07\$05.00/0.

© 1991 Optical Society of America.

been shown.<sup>8</sup> The possibility of the creation of squeezed states of optical fields has been shown,<sup>9</sup> and a high degree of squeezing has been reached in practice.<sup>10</sup> Using this effect provides good opportunities to create Einstein-Podolsky-Rosen paradox conditions,<sup>11</sup> to test Bell's inequality,<sup>12,13</sup> and to investigate electromagnetic vacuum behavior in a nonlinear sample.<sup>14</sup>

The first experiments with SPSSL<sup>15-19</sup> showed the existence of strong correlations of photons in space and time that distinguish this effect from other processes of light scattering. The methods of one-photon state localization in time only<sup>20</sup> and in both space and in time<sup>21</sup> were developed with the SPSSL process. Using this property of the SPSSL, we developed a new method of measuring the spectral distribution of the absolute value of the quantum efficiency of photodetectors discussed in this paper. This method does not require the use of conventional metrological calibrated standard light sources (étalons). It has, from our point of view, great promise for widespread use and can be incorporated into conventional optical metrology.

In this paper we discuss the main results and details of the absolute standardless calibration technique. The SPSSL makes the best calibrations for photodetectors in the photon-counting regime because when this method is used the nonclassical properties of light in the SPSSL process are revealed with better contrast for single photons. The photon-counting method is widely used to measure low level optical signals, especially in astronomy and spectroscopy. We briefly describe the theoretical background of this photon-counting technique and then present results of the practical realization of this new method for several types of photomultiplier tube in the photon-counting regime.

Many contemporary photodetectors work in the analog (dc) regime, which often has some advantages over the photon-counting regime. We have developed a method of absolute measurement of quantum efficiency for the broad class of photodetectors working in the analog (charge-accumulation) regime. The present paper contains this result, also. In our conclusion, we discuss the status of present developments and improvements of proposed methods.

## Theory

The method of absolute calibration of photodetectors is based on the rigorous space and time coherence of photons in the SPSSL effect. We refer here to the supercorrelation of photons (compared with the classical Hanbury Brown-Twiss correlation for thermal light sources) and to the existence of a biphotonic (two-photon) optical field. The theory of this method was proposed and developed by Klyshko.<sup>2</sup> Here we discuss only the basic idea necessary for interpretation of experimental results. The first realization was carried out with photomultiplier tubes in the photon-counting regime.<sup>16</sup> Recently some experiments were made and results obtained by using this method for the calibration of avalanche photodiodes in the Geiger-mode regime at one spectral point.<sup>12,22</sup>

Let pump radiation of frequency  $\omega_0$  and wave vector  $\mathbf{k}_0$  be directed toward the nonlinear crystal with a nonzero value of second-order nonlinear susceptibility  $\chi^2$ . In each scattering event a pump photon is converted into a pair of photons with frequencies  $\omega_1$  and  $\omega_2$  and wave vectors  $\mathbf{k}_1$  and  $\mathbf{k}_2$ . The probability of this effect has maximum value in directions determined by the phase-matching conditions

$$\omega_1 + \omega_2 = \omega_0, \quad \mathbf{k}_1 + \mathbf{k}_2 = \mathbf{k}_0. \quad (1)$$

This distribution of radiation in space depends on the dispersion of the refractive index of the nonlinear sample and on the mutual orientation of the pump wave vector  $\mathbf{k}_0$  and the optical axis of the crystal ( $\mathbf{z}$ ).

The nonclassical character of the scattered radiation can be revealed with the best contrast in the photon-counting regime by the correlation technique. The statistics of photocurrent pulses have been thoroughly investigated.<sup>23-26</sup> The probability of the creation of a photocurrent pulse in the ideal photodetector is defined by<sup>27</sup>

$$P_i = \frac{c}{2\pi} \eta_i \int dA_i \int_t^{t+T} dt' \langle E_i^{(-)} E_i^{(+)} \rangle, \quad i = 1, 2, \quad (2)$$

where  $\eta_i$  is the quantum efficiency of the photodetector,  $A$  is the photodetector area,  $T$  is the interval of measurement in time, and  $E_i^{(-)}$  and  $E_i^{(+)}$  are the negative and positive frequency components, respectively, of electromagnetic fields of frequency  $\omega_i$ .

The probability of photocurrent coincident pulses for two ideal photodetectors is then<sup>27</sup>

$$P_c = \frac{c^2}{4\pi^2} \eta_1 \eta_2 \int dA_1 \int dA_2 \int_0^T dt_1 \int_\tau^{t_1+T} dt_2 \langle E_1^{(-)} E_2^{(-)} E_1^{(+)} E_2^{(+)} \rangle, \quad (3)$$

where  $\tau$  is a delay time between two channels.

In the SPSSL process the probability that the photocurrent pulse will emerge in one detector is defined by<sup>2,5</sup>

$$P_1 = \sum_{n_1} \eta_1 \langle N_1 \rangle, \quad (4)$$

where  $n_1$  enumerates the field modes inside the detection volume and  $\langle N_1 \rangle = \langle a_1^\dagger a_1 \rangle$  is the mean number of photons in a mode of scattered radiation of frequency  $\omega_1$ . The same is true for the second photodetector that registers photons of frequency  $\omega_2$ .

The probability of the photocurrent coincident pulses in our case is

$$P_c = \sum_{n_1 n_2} \eta_1 \eta_2 \langle N_1 N_2 \rangle. \quad (5)$$

It can easily be shown<sup>2,5</sup> that, as a result of the nonclassical rigorous correlation of pairs of photons in the SPSSL process, in the case of coherent pump radiation in the absence of losses we will have

$$\langle N_1 \rangle = \langle N_2 \rangle = \langle N_1 N_2 \rangle. \quad (6)$$

Thus, if the photodetector to be calibrated (detector 1) is capturing all spectral and angular modes of the

optical field parametrically conjugated by the phase-matching condition [Eqs. (1)] with modes registered by detector 2 (the reference photodetector), and if for these modes  $\eta_2$  is constant (usually  $\Delta\omega_1 \ll \Delta\omega_2$ ), then from Eqs. (4)–(6) we can obtain the value of the quantum efficiency of detector 1:

$$\eta_1 = P_c/P_2. \quad (7)$$

Hence a simultaneous measurement of three quantities,  $P_c$ ,  $P_1$ , and  $P_2$ , allows one to determine quantum efficiency for both detectors because all these results are symmetric with respect to interchange of index labels  $1 \leftrightarrow 2$ .

The result of the measurement can be expressed both in percentage form and in natural absolute units (electrons/photon or counts/photon).

### Absolute Calibration of Photon-Counting Detectors

A schematic of the experimental setup is shown in Fig. 1. The apparatus consists of two parts: a biphoton field generator and a two-channel correlation scheme for the registration and processing of the signal. The biphoton field generator includes an ultraviolet radiation source (a cw He–Cd laser with average power of  $\sim 5$  mW at a 325-nm wavelength), a parametric downconverter (a 1 cm  $\times$  1 cm  $\times$  1 cm cube of LiIO<sub>3</sub> crystal), and a lens with a focal length  $f = 100$  mm and relative aperture 1:3.5. Such a generator provides a flux of biphotons (photon pairs) with continuous spectral distribution from 400 to 1000 nm with a density of  $10^8$  pairs per second in a spectral range of  $\Delta\nu = 10$  cm<sup>-1</sup>.<sup>28</sup> The scattered radiation in the lens's focal plane consists of a sequence of concentric colored rings. The ring thickness is defined by the value of the spatial coherence of radiation and by the value of the focal length.<sup>2</sup> Figure 2 shows a group of  $\theta_1(\lambda_1)$  [ $\theta_2(\lambda_2)$ ] curves that define the distribution of scattered radiation in space (in the focal plane) depending on its frequency. The angle  $\varphi$  between  $\mathbf{k}_0$  and the crystal optic axis is a variable parameter. The case  $\omega_1 = \omega_2 = (\omega_0/2)$  corresponds to the frequency-degenerate regime of photon pair generation. Continuous spectral and smooth angular distribution of scattered radiation

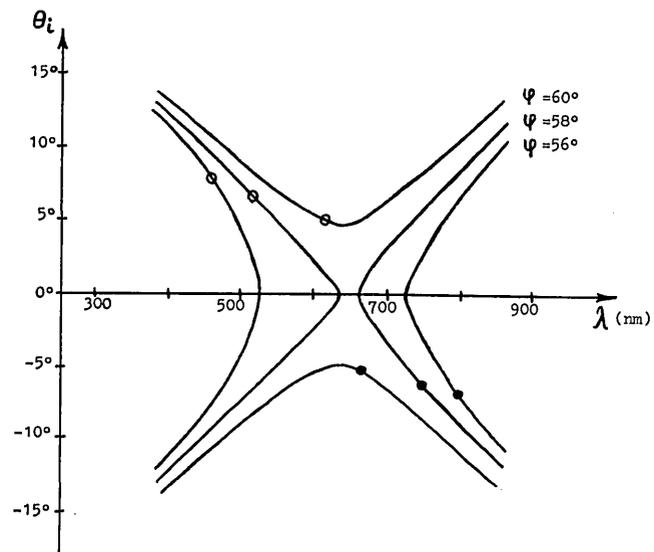


Fig. 2. Spectral-angular intensity distribution of light emerging from the LiIO<sub>3</sub> nonlinear crystal pumped by radiation with wavelength  $\lambda_0 = 325$  nm. Here  $\theta_i = (\mathbf{k}_i, \mathbf{k}_0)$  and  $\varphi$  is the angle between the pump wave vector  $\mathbf{k}_0$  and the optical axis of crystal ( $z$ ); photons conjugated by condition (1) are depicted by ● and ○.

(see Fig. 2) is convenient for the absolute calibration of photodetectors. Conjugated by the phase-matching conditions [Eq. (1)], photons in each pair (denoted ● and ○ in Fig. 2) are divided into two different channels and counted by the photodetectors (4). In the measuring channel (1), photons pass only through the exit border of the crystal and the lens. The lens transforms angular dependences  $\theta_{1,2}(\omega)$  into linear dependences  $r_{1,2}(\omega)$  in the focal plane. This element of the scheme is not necessary; however, it greatly simplifies the procedure of positioning and provides a better filtration of biphotons with a given relation between frequencies and wave vectors. The spectral device (3) placed in the reference channel (2) is used to measure the spectral distribution of the quantum-efficiency value  $\eta_1(\lambda)$ .

The electric signal-processing and registration scheme consists of two identical channels amplifying

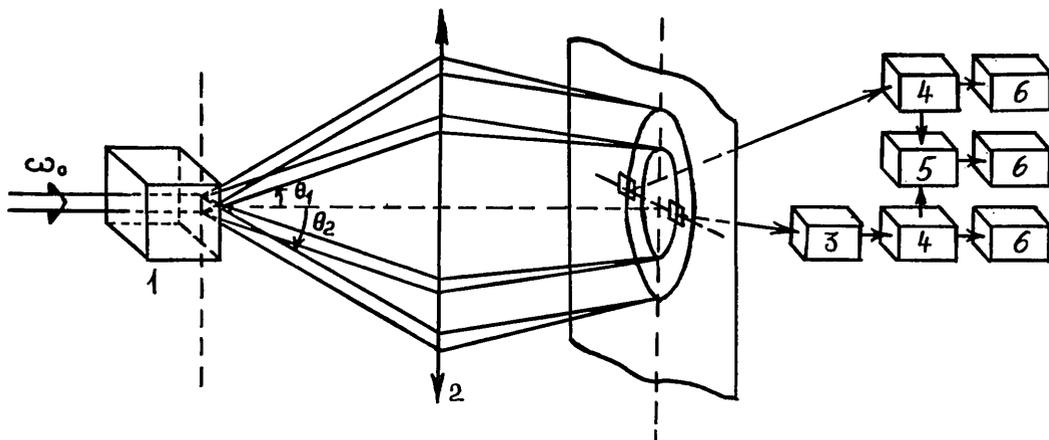


Fig. 1. Schematic of the experimental setup for the absolute standardless calibration of photodetectors: 1, nonlinear crystal; 2, focusing lens; 3, spectral device; 4's, photodetectors; 5, coincident count scheme, 6's, counters.

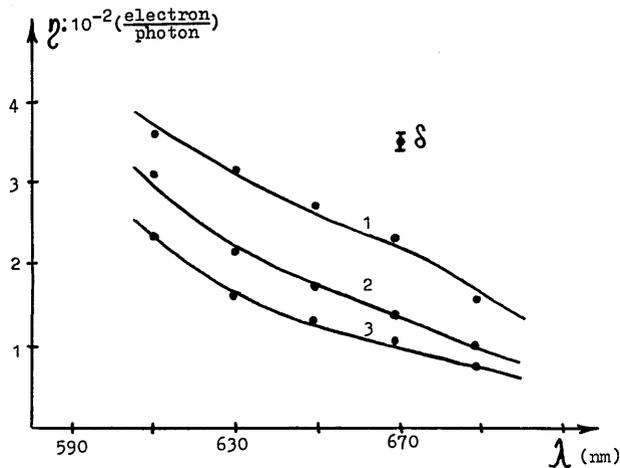


Fig. 3. Quantum-efficiency spectral distribution for three samples of a Soviet photomultiplier tube, type FEU-79 (multialkali S-20 photocathode). The integral sensitivity  $S_{\Sigma}$  of devices 1, 2, and 3 was 306, 210, and 143  $\mu\text{A}/\text{lm}$ , respectively.

and forming the photocurrent pulses from the detectors, a coincidence scheme, and three devices for data storage and indication. Electrical pulses after the photodetectors (4) and amplifiers have a rise time of  $\tau = 3$  ns. They are fed to the coincidence counting scheme (5) and to the data accumulator (6). The time resolution of the registration scheme, i.e., the maximum interval between the entrance two pulses, was  $< 5 \times 10^{-9}$  s. The dynamic linear range for the counting frequency was  $> 10^6$   $\text{s}^{-1}$ . The density of the two-photon flux was made so weak that the time resolution of the electronic scheme was much higher than the average time between photon pairs to reduce the influence of dead time on the result of measurements. Real photon pair flux did not exceed  $10^5$   $\text{s}^{-1}$ .

Our research shows the reliability of the principles and the method of this new type of calibration scheme. In particular, we observed good recurrence properties and small sensitivity to variation of experimental parameters. For example, the  $\eta_1$  value remains constant, with an accuracy no worse than 1%, when the pump power value is changed by 2 orders of magnitude. We also observed no influence on the result of the quantum efficiency  $\eta_1$  measurement from the insertion of a new spectral device, from the insertion of additional sources of losses in the reference channel, or from the replacement of the reference detector (detector 2) by another having different parameters.

We measured the quantum efficiency of several types of photomultiplier tube in the photon-counting regime. The statistical error of the measurements did not exceed 1–3% with the time of data accumulation at  $T = 100$  s. We measured the quantum efficiency of photomultiplier tubes with both multialkali (S-20) and bialkali photocathodes, with different numbers of diodes and their configuration, and with different time properties. We show the results of quantum-efficiency spectral distribution measurements of a few samples of Soviet photomultiplier tubes: FEU-79 in Fig. 3, fast FEU-77 in Fig. 4, and quantacons FEU-130 and

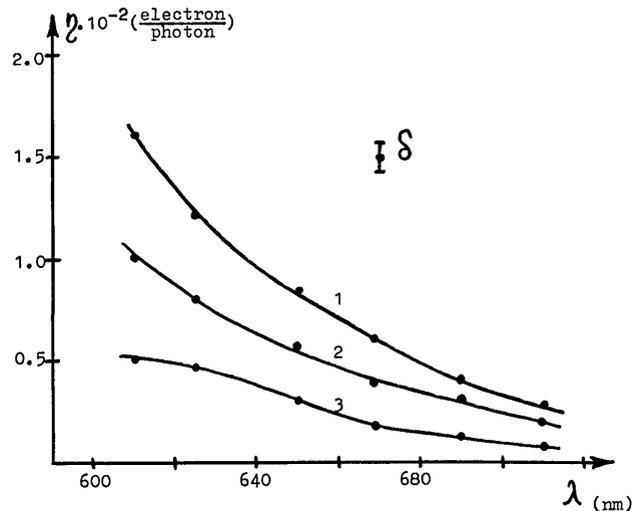


Fig. 4. Spectral distribution of quantum efficiency for three samples of a Soviet fast photomultiplier tube, type FEU-77 (S-20). The integral sensitivity  $S_{\Sigma}$  of devices 1, 2, and 3 was 220, 160, and 78  $\mu\text{A}/\text{lm}$ , respectively.

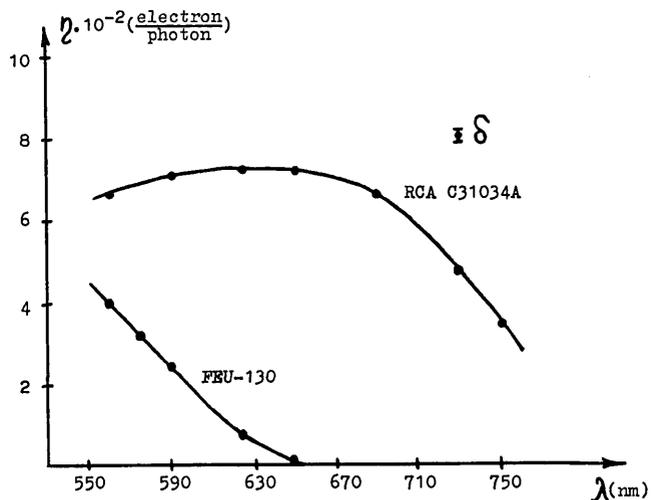


Fig. 5. Spectral distribution of quantum efficiency for RCA C31034A (GaAs photocathode) and FEU-130 (bi-alkali photocathode) quantacons.  $S_{\Sigma} = 60$   $\mu\text{A}/\text{lm}$ .

RCA-C31034A in Fig. 5.  $S_{\Sigma}$  is a value of the integral cathode sensitivity expressed in (microamperes per lumen) and usually published in the datasheet of photodetectors.

We compared several results of quantum-efficiency absolute measurements with data received by the conventional method of photometry.<sup>2</sup> All results coincided well within the limits of statistical error. It is noteworthy that the accuracy of conventional methods based on the use of secondary light étalons (incandescent lamps with tungsten bands that imitate radiation of an absolute blackbody) is  $\sim 10$ – $15\%$  in this spectral range and for these intensities.

#### Absolute Calibration of Analog Photodetectors

The photodetectors working in the analog (charge-accumulation) regime are widely used now for a variety

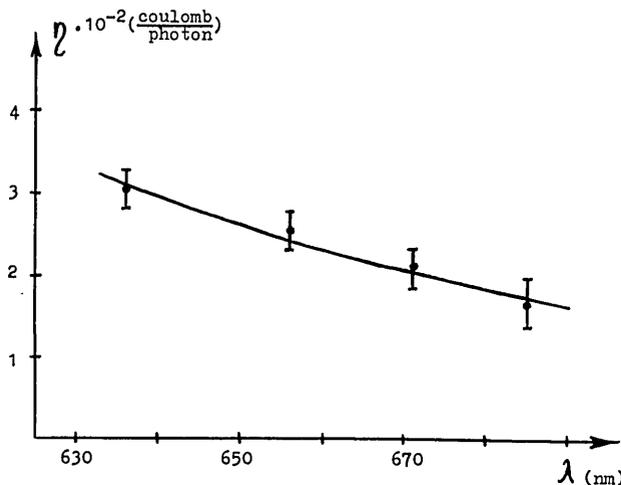


Fig. 6. Spectral distribution of quantum efficiency for photomultiplier tube FEU-79 (multialkali photocathode  $S_{\Sigma} = 238 \mu\text{A/lm}$ ) in the analog regime of charge accumulation.

of purposes. In some cases, if one analyzes the combined sources of errors for both methods (the level of the inherent dark noise, the absolute value of quantum efficiency, the dead-time value, etc.), they have advantages over the photon-counting devices.<sup>29,30</sup> An optical signal intensity  $I_0$  exists for which registrations by both methods are of equal accuracy. When the intensity of the optical signal is higher than  $I_0$ , the charge-accumulation method is more advantageous from the point of view of the signal-to-noise ratio, and in the opposite case—for an intensity lower than  $I_0$ —the photon-counting method is favored.

We also developed a method of absolute calibration of analog photodetectors. For our first experiments, we used a photomultiplier tube in the classical charge accumulation regime. The same optical scheme as shown in Fig. 1 was used. The practical realization of this method is different in several important details from the method for the photon-counting devices. The information about the biphotonic character of radiation reaching the photocathode is now carried by the continuous photocurrent fluctuation. The electrical signal after the photodetector can be expressed as a sequence of overlapped photocurrent pulses:

$$i(t) = \sum_n q_n f_n(t - t_n). \quad (8)$$

Here  $f(t)$  describes the shape of pulses with random amplitude  $q_n$  and random moment of emergence  $t_n$ . The theoretical analysis shows<sup>2</sup> that, if the shape of pulses in both channels is the same [ $f_1(t) = f_2(t)$ ], then, by analogy with the photon-counting calibration method [Eqs. (4)–(7)], we can derive the following expression for the quantum-efficiency value:

$$\eta_1 \langle q_1 \rangle = \xi_2 \langle q_2 \rangle \frac{\langle i_1 i_2 \rangle}{\langle i_2^2 \rangle}. \quad (9)$$

Here  $\langle q_i \rangle$  is the mean value of the random pulse amplitude in each channel, and  $\xi_i = \langle q_i^2 \rangle / \langle q_i \rangle^2$  is the coefficient dependent on the statistics of amplification coef-

ficient fluctuations. The coefficient  $\xi_i$  determines the difference between the amplitude distribution of photocurrent pulses and the Poissonian one. These parameters can be measured easily with good accuracy.<sup>31</sup> The measured value of quantum efficiency can be expressed either as a percentage or in absolute natural units (coulombs/photon).

The procedure  $\langle i_1 i_2 \rangle$  was carried out by an analog current multiplier device with 1% linearity in the dynamic range from 10 mV to 10 V. The output signal was fed to an analog-to-digital converter, whose output was accumulated by a counter. The results of quantum-efficiency measurements of a photomultiplier tube with multialkali (S-20) photocathodes in the charge-accumulation regime are shown in Fig. 6. The statistical accuracy of the measurements was ~6–8%. The main contribution to this error was the statistical fluctuations of the electrical signal in the charge accumulation regime, which are  $2(1 + \xi)$  times greater than in the photon-counting regime in identical conditions of measurement because of the non-Poissonian distribution of photocurrent pulse amplitudes.<sup>29</sup> However, this problem can easily be eliminated by increasing the signal-accumulation time.

## Conclusion

An experimental realization of a new way to measure the absolute value of quantum efficiency for photon-counting and analog photodetectors without using any specially calibrated light sources (étalons) has been discussed. This technique is based on the nonclassical statistical properties of light in the nonlinear process of spontaneous parametric scattering of light (spontaneous parametric downconversion). The results of our experiments provide a hope that this new method of standardless calibration of photodetectors can be widely used, especially to extend the capability of conventional photometry.

The investigation of the influence of different physical processes on the results of calibration allows us to speak about the possibility of reaching 0.1% and greater accuracy in measurements in photon-counting regime devices. The quantum efficiency of photomultiplier tubes (both ordinary and microchannel) or of avalanche photodiodes can be measured in this regime. The measurements of the quantum efficiency of avalanche photodiodes at one spectral point (532 nm in Ref. 12 and 650 nm in Ref. 22) carried out by other authors confirm the reliability of this method in the photon-counting regime. Thus a new method provides an easy way to measure in the laboratory the spectral distribution of the quantum-efficiency value for each sample of photomultiplier tubes from a manufactured batch. This measurement is useful when one wishes to choose a suitable detector for the registration of ultralow-intensity optical signals, especially in astronomy and spectroscopy.

We propose combining the apparatus for photodetector calibration by biphoton fields and by a high-efficiency self-calibration photodiode device to build an absolute radiometer with a broad dynamic range.

For example, from Ref. 1, a photodiode device with 100% quantum efficiency is characterized by a high noise level—up to  $10^{-8}$  A. Such a level of noise limits the measured intensities to the level of  $10^{11}$  photons per second (for the center of the visible spectrum it is  $10^{-8}$  W). The noise of photon counters (for example, photomultiplier tubes) is much lower,  $\sim 1$ – $10$  photocurrent pulses per second, which is equivalent (if the current amplification coefficient is  $10^6$ ) to the photocurrent  $10^{-13}$  A. For the quantum-efficiency value  $\eta = 10\%$ , such a noise level guarantees the registration of photon fluxes down to  $10^{-17}$ – $10^{-18}$  W.

The analog calibration method for devices that work in a classical current regime, despite the higher statistical error in equal conditions, has a broad range of uses. The main advantage of the analog method of photodetector calibration is its broad application. It is possible, based on this method, to calibrate practically all types of contemporary photodetector, including the photon-counting devices, in a wide spectral range (the reverse is not true for the method of photon-counting detector calibration). It can be used for the calibration of spectral sensitivity for practically all kinds of photodetector, such as photomultiplier tubes, avalanche and ordinary photodiodes, multilayer photosensitive semiconductor structures, and charge-coupling devices, in broad spectral and dynamic ranges. These broad capabilities are based on the ability to create biphotonic (two-photon) fields with the required intensity in the extended spectral range from 0.35 to  $5 \mu\text{m}$  with the help of the SPSL process. The accuracy of the analog calibration method can be improved easily by cooling the photodetector and increasing the data-accumulation time.

The rigorous space localization of photons in pairs (see Figs. 1 and 2) also permits development of a method of absolute measurement of the two-dimensional picture of absolute sensitivity of image-intensifier devices such as a silicon-intensified target and charge-coupled device cameras. For the matrix photodetectors, calibration of each pixel is possible in principle.

It should be mentioned here that, all other things being equal, the new method works better (higher accuracy, shorter time of measurement, etc.) the higher the absolute value of the quantum efficiency.

We have shown in our experiments the feasibility of a new method. Many problems relating to the utilization of biphoton fields in metrology still remain unsolved; however, the utility of the method is obvious. It will be useful to compare our results with the results of measurements done by well-known metrology techniques including the precise thermal calorimeter and silicon photodiode self-calibration methods. We hope to make further developments and to improve the accuracy of our method in collaboration with other organizations interested in this problem.

## References

1. E. F. Zalewski and J. Geist, "Silicon photodiode absolute spectral response self-calibration," *Appl. Opt.* **19**, 1214–1219 (1980); E. F. Zalewski and C. R. Duda, "Silicon photodiode device with 100% external quantum efficiency," *Appl. Opt.* **22**, 2867–2871 (1983).
2. D. N. Klyshko, "Use of vacuum fluctuations as an optical brightness standard," *Sov. J. Quantum Electron.* **7**, 591–599 (1977); "Utilisation of a two-photon light for absolute calibration of photoelectric detectors," *Sov. J. Quantum Electron.* **10**, 1112–1117 (1980).
3. D. N. Klyshko, "Coherent photon decay in a nonlinear medium," *Sov. Phys. JETP Lett.* **6**, 23–25 (1967); S. E. Harris, M. K. Oshman, and R. L. Beyer, "Observation of tunable parametric fluorescence," *Phys. Rev. Lett.* **18**, 732–735 (1967); O. Magde and H. Mahr, "Study in ammonium dihydrogen phosphate of spontaneous parametric interaction tunable from 400 to 16 000 Å," *Phys. Rev. Lett.* **18**, 905–912 (1967).
4. T. G. Giallorenzi and C. L. Tang, "Quantum theory of spontaneous parametric scattering of intense light," *Phys. Rev.* **166**, 905–912 (1968); D. N. Klyshko, "Scattering of light in a medium with nonlinear polarizability," *Sov. Phys. JETP* **28**, 522–527 (1969).
5. D. N. Klyshko, *Photons and Nonlinear Optics* (Nauka, Moscow, 1980; Gordon & Breach, New York, 1988).
6. Some of the earliest statistical treatments were by B. R. Mollow and R. J. Glauber, "Quantum theory of parametric amplification," *Phys. Rev.* **160**, 1076–1094 (1967); B. R. Mollow, "Quantum statistics of coupled oscillator systems," *Phys. Rev.* **162**, 1256 (1967); D. A. Kleinman, "Theory of optical parametric noise," *Phys. Rev.* **174**, 1027–1039 (1968); B. Y. Zeldovich and D. N. Klyshko, "Statistics of field in parametric luminescence," *Sov. Phys. JETP Lett.* **9**, 40–42 (1969); J. Tucker and D. F. Walls, "Quantum theory of the traveling-wave frequency converter," *Phys. Rev. Lett.* **178**, 2036–2043 (1968); B. R. Mollow, "Photon correlation in the parametric frequency converter," *Phys. Rev. A* **8**, 2684–2698 (1973).
7. R. Graham, "General correlation identity," *Phys. Rev. Lett.* **52**, 117–121 (1984); C. K. Hong and L. Mandel, "Theory of parametric frequency down-conversion of light," *Phys. Rev. A* **31**, 2409–2438 (1985).
8. E. Jakeman and J. G. Walker, "Analysis of a method for the generation of light with sub-Poissonian photon statistics," *Opt. Commun.* **55**, 219–223 (1985); J. G. Rarity, P. R. Tapster, and E. Jakeman, "Observation of sub-Poissonian light in parametric downconversion," *Opt. Commun.* **62**, 201–205 (1987).
9. S. F. Friberg and L. Mandel, "Production of squeezed states by combination of parametric down-conversion and harmonic generation," *Opt. Commun.* **48**, 439–442 (1984).
10. Ling-An Wu, H. J. Kimble, J. L. Hall and H. Wu, "Generation of squeezed states by parametric down conversion," *Phys. Rev. Lett.* **57**, 2520–2524 (1986).
11. D. N. Klyshko, "The Einstein–Podolsky–Rosen paradox for energy–time variables," *Sov. Phys. Usp.* **32**, 555–562 (1989).
12. Y. H. Shih and C. O. Alley, "New type of Einstein–Podolsky–Rosen–Bohm experiment using pairs of light quanta produced by optical parametric down conversion," *Phys. Rev. Lett.* **61**, 2921–2925 (1988).
13. R. Chosh and L. Mandel, "Observation of nonclassical effects in the interference of two photons," *Phys. Rev. Lett.* **59**, 1903–1907 (1987).
14. G. K. Kitaeva, A. N. Penin, and A. V. Sergienko, "Interference of zero-point fluctuations of the electromagnetic vacuum and photon correlation in parametric light scattering," *Sov. Phys. Dokl.* **32**, 293–295 (1987).
15. D. C. Burnham and D. L. Weinberg, "Observation of simultaneity in parametric production of photon pairs," *Phys. Rev. Lett.* **25**, 84–87 (1970).
16. A. A. Malygin, A. N. Penin, and A. V. Sergienko, "Absolute calibration of the sensitivity of photodetectors using biphotonic field," *Sov. Phys. JETP Lett.* **33**, 477–481 (1981).
17. S. Friberg, C. K. Hong, and L. Mandel, "Measurement of time delays in the parametric production of photon pairs," *Phys. Rev. Lett.* **54**, 2011–2015 (1985).

18. I. Abram, R. K. Raj, J. L. Oudar, and G. Doligne, "Direct observation of the second order coherency of parametrically generated light," *Phys. Rev. Lett.* **57**, 2516-2521 (1986).
  19. C. K. Hong, L. Mandel, and Z. Y. Ou, "Measurement of subpicosecond time intervals between two photons," *Phys. Rev. Lett.* **59**, 2044-2049 (1987).
  20. C. K. Hong and L. Mandel, "Experimental realization of a localized one-photon state," *Phys. Rev. Lett.* **56**, 58-61 (1986).
  21. A. A. Malygin, A. N. Penin, and A. V. Sergienko, "Spatio-temporal grouping of photons in spontaneous parametric scattering of light," *Sov. Phys. Dokl.* **30**, 227-231 (1985).
  22. J. G. Rarity, K. D. Ridley, and P. R. Tapster, "Absolute measurement of detector quantum efficiency using parametric downconversion," *Appl. Opt.* **26**, 4616 (1987).
  23. L. Mandel, "Fluctuations of photon beams and their correlations," *Proc. Phys. Soc. London* **72**, 1037-1045 (1957).
  24. H. P. Yuen, J. H. Shapiro, and J. A. Machado Mata, "Optical communication with two-photon coherent states—Part II: Photoemissive detection and structured receiver performance," *IEEE Trans. Inf. Theor.* **IT-25**, 179-191 (1979).
  25. D. F. Walls, G. J. Milburn, and H. J. Carmichael, "Photoelectron counting probabilities for non-classical fields," *Opt. Acta* **29**, 1179-1198 (1982).
  26. H. J. Kimble and L. Mandel, "Photoelectron detection of polychromatic light," *Phys. Rev. A* **30**, 844-853 (1984).
  27. R. J. Glauber, "Optical coherence and photon statistics," in *Quantum Optics and Electronics*, C. de Witt, A. Blandin, and C. Cohen-Tannoudji, eds. (Gordon & Breach, New York, 1965).
  28. A. A. Malygin, A. N. Penin, and A. V. Sergienko, "Efficient generator of two-photon field in the Visible region," *Sov. J. Quantum Electron.* **11**, 939-941 (1981).
  29. R. Jones, C. J. Oliver, and E. R. Pike, "Experimental and theoretical comparison of photon counting and current measurements of light intensity," *Appl. Opt.* **10**, 1673-1687 (1971).
  30. V. R. Astafurov and G. N. Glasov, "Comparison of accuracy in light intensity measurement by photon counting and charge accumulation methods," *Radiokh. Elektron.* **1**, 148-156 (1985).
  31. A. V. Sergienko and A. N. Penin, "Absolute calibration of analog photodetectors using biphoton fields," *Sov. Tekh. Phys. Lett.* **12**, 328-330 (1986).
-