



localized about an arbitrary position  $\xi$  in the nonlinear crystal [9]:

$$|\Psi_{\xi}^{(2)}\rangle \sim |0\rangle - \frac{i}{\hbar} \int_{t_0}^t dt' \chi^{(2)} \int_{-L}^0 dz W(z - \xi) \\ \times [E_p^{(+)}(t', z) \mathbf{e}_s \hat{E}_s^{(-)}(t', z) \mathbf{e}_i \hat{E}_i^{(-)}(t', z) + \text{H.c.}] |0\rangle. \quad (1)$$

Here  $\chi^{(2)}$  is the second-order susceptibility of the nonlinear medium;  $\mathbf{e}_j$  is the unit vector denoting the polarization state of the  $j$ th field  $\hat{E}_j^{(-)}(t', z)$ ; the subscripts  $p$ ,  $s$ , and  $i$  represent the pump, signal, and idler, respectively; the pump field is assumed to be a propagating classical transform-limited pulse whose amplitude is slowly varying:

$$E_p^{(+)}(t', z) = \sqrt{I_p \left( t' - \frac{z}{v_p} \right)} \exp[i(k_p^0 z - \omega_p^0 t')]. \quad (2)$$

The quantity  $I_p(t', z)$  repeats the intensity profile of the pump at time  $t'$  and position  $z$  along the propagation axis with a full width at half maximum (FWHM) denoted  $\tau_p$ . Because of the characteristic of our laser beam, we specifically chose it to have a Gaussian form,  $\sqrt{I_p(t', z)} = \sqrt{I_0} \exp[-(t' - \frac{z}{v_p})^2 / 2\sigma^2]$  with  $\sigma = \tau_p / (2\sqrt{\ln 2})$ , though the theory is more general and can accommodate any pulse shape. The central wave number, carrier frequency, and group velocity of the pump (at the carrier frequency inside the nonlinear crystal) are denoted in Eq. (2) by  $k_p^0$ ,  $\omega_p^0$ , and  $v_p$ , respectively. The explicit form of  $W(z - \xi)$  is taken to be a half-Gaussian with argument  $(z - \xi)$  and width  $d_{\text{eff}}$ ; its magnitude decreases with increasing argument as a consequence of the effective gradual separation of the pump pulse from the down-converted fields in the nonlinear crystal arising from linear dispersion. This function reflects the distinguishability not only between the down-converted photons of each pair but also between pairs born at different locations as the femtosecond pump pulse traverses the nonlinear crystal.

We now consider the timing diagram shown in Fig. 2 in conjunction with the schematic of our experimental apparatus presented in Fig. 1. Once an orthogonally polarized photon pair in the laboratory polarization basis [ $H$  (horizontal),  $V$  (vertical)] is generated at time  $t_0$  around position  $\xi$ , it propagates through the nonlinear down-conversion crystal, acquiring a relative time delay  $\phi_{\xi}$  imparted by the birefringence of the nonlinear crystal. The two-photon state in the  $(H, V)$  basis,

$$|\Psi_{\xi}^{(2)}\rangle \sim |V\rangle_{t_0} |H\rangle_{t_0 + \phi_{\xi}}, \quad (3)$$

then propagates through a polarization delay line made of birefringent material (such as crystalline quartz) oriented so that its fast  $X$  (slow  $Y$ ) axis is rotated by  $45^\circ$  with respect to the  $H$  ( $V$ ) polarization axis (projection and optical-path-delay operations are displayed separately in

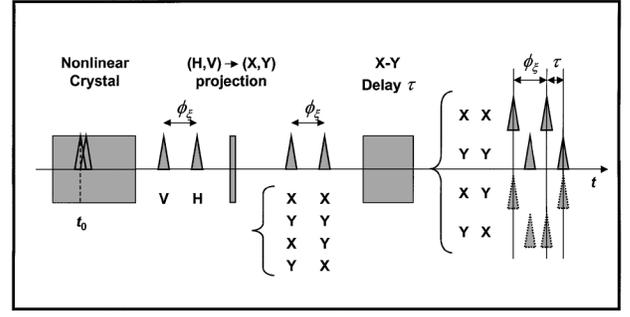


FIG. 2. A cross-polarized photon pair generated in the vicinity of position  $\xi$  within the nonlinear down-conversion crystal acquires a relative time delay  $\phi_{\xi}$ . These photons are projected to a new basis  $(X, Y)$  that is tilted by  $45^\circ$  with respect to the original basis  $(H, V)$ , and the  $Y$ -projected photon experiences a relative optical-path delay  $\tau$  with respect to the  $X$ -projected photon as a result of the birefringence of the delay element. (Projection and optical-path delay operations are displayed separately for clarity.) Although there appear to be four possible configurations that the photon pair can assume before arriving at the beam splitter (far right), the bottom two (dotted) vanish because of their coefficients. The remaining two contributions (solid) preserve their relative delay  $\phi_{\xi}$ , so that the interfering terms are the  $XX$  and  $YY$  terms with a delay  $\tau$ , whatever the value of  $\phi_{\xi}$ .

Fig. 2 for clarity). As a result of the projection, the two-photon state in the rectilinear  $(X, Y)$  basis becomes

$$|\Psi_{\xi}^{(2)}\rangle \sim (|X\rangle_{t_0} + |Y\rangle_{t_0}) (|X\rangle_{t_0 + \phi_{\xi}} - |Y\rangle_{t_0 + \phi_{\xi}}). \quad (4)$$

Finally, the longitudinal optical-path delay  $\tau$  along the  $Y$  axis with respect to the  $X$  axis gives rise to

$$|\Psi_{\xi}^{(2)}\rangle \sim (|X\rangle_{t_0} |X\rangle_{t_0 + \phi_{\xi}} - |Y\rangle_{t_0 + \tau} |Y\rangle_{t_0 + \phi_{\xi} + \tau}) \\ + (|Y\rangle_{t_0 + \tau} |X\rangle_{t_0 + \phi_{\xi}} - |X\rangle_{t_0} |Y\rangle_{t_0 + \phi_{\xi} + \tau}), \quad (5)$$

which is illustrated at the far right of Fig. 2. This then arrives at a nonpolarizing beam splitter, as shown in Fig. 1, which injects photons into the two arms of the polarization interferometer (denoted in the following equations as subscripts 1 and 2). The polarization analyzers at the ends of the arms, being the final elements in the polarization interferometer, transform the two-photon state once more, via the operator  $\hat{P} \equiv |\Theta\rangle_1 |\Theta\rangle_2 \langle\Theta|_1 \langle\Theta|_2$ , which is constructed in the polarization analyzer states  $\Theta_1$  and  $\Theta_2$ .

Taking account of the transformations described above, one can calculate the two-photon detection probability amplitude

$$\mathcal{A}_{\xi}(t_1, t_2) = \langle 0 | \hat{E}_1^{(+)} \hat{E}_2^{(+)} | \Psi_{\xi}^{(2)} \rangle, \quad (6)$$

where  $t_1$  ( $t_2$ ) denotes the event registration time at the detector located at the first (second) arm of the interferometer. For an arbitrary region about the position  $\xi$  in the crystal, Eq. (6) takes the explicit form [9]

$$A_{\xi}(T, t) \equiv \mathcal{A}_{\xi}(t_1, t_2) \sim \sqrt{I_p \left( T - \frac{\Lambda}{D} t - \frac{\xi}{v_p} \right)} \\ \times W(t) \text{rect}_{[-L, 0]} \left( \frac{t}{D} + \xi \right) \\ \times \exp[i(k_p^0 \xi - \omega_p^0 T)], \quad (7)$$

where  $T = (t_1 + t_2)/2$ ,  $t = (t_1 - t_2)$ ,  $\Lambda = [1/v_p - (1/2)(1/v_s + 1/v_i)]$ , and  $D = (1/v_i - 1/v_s)$ . The function  $\text{rect}_{[-L,0]}(z)$  has a unity value when its argument is in the range  $[-L, 0]$  and vanishes otherwise, and  $W(t)$  is the partial decoherence function discussed above.

When we carry out the above-described set of transformations, the rotated optical delay line causes multiple possibilities of coincidence detection, thereby yielding an overall two-photon detection probability amplitude that is the superposition of these possibilities. However, as a consequence of the cascaded transformations, the possibilities originating from the two right-most terms in Eq. (5) (shown as dotted in Fig. 2) vanish, leaving only two surviving terms that contribute to the overall two-photon amplitude:

$$B_{\xi,\tau}(T, t) \sim \cos\left(\frac{\pi}{4} - \theta_1\right) \cos\left(\frac{\pi}{4} - \theta_2\right) A_{\xi}(T, t) - \sin\left(\frac{\pi}{4} - \theta_1\right) \sin\left(\frac{\pi}{4} - \theta_2\right) \times A_{\xi}(T + \tau, t). \quad (8)$$

Finally, using Eq. (8) in the coincidence-count rate equation [1] provides

$$R(\tau) = \int_{-L+d_{\text{eff}}}^0 d\xi \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} dT dt |B_{\xi,\tau}(T, t)|^2, \quad (9)$$

from which we determine the explicit form of the coincidence-count rate at the detectors, as a function of polarization-analyzer angles  $(\theta_1, \theta_2)$  and optical-path relative delay time  $(\tau)$ , to be

$$R(\theta_1, \theta_2, \tau) = G_p(0) \int_{-L+d_{\text{eff}}}^0 d\xi \Phi(\xi) \left[ (\cos^2 \vartheta_1 \cos^2 \vartheta_2 + \sin^2 \vartheta_1 \sin^2 \vartheta_2) - 2(\sin \vartheta_1 \sin \vartheta_2 \cos \vartheta_1 \cos \vartheta_2) \frac{G_p(\tau)}{G_p(0)} \cos(\omega_p^0 \tau) \right], \quad (10)$$

where  $G_p(\tau) = \int_{-\infty}^{\infty} dt \sqrt{I_p(t)I_p(t + \tau)}$  and  $\vartheta_i \equiv \pi/4 - \theta_i$ ,  $i = 1, 2$ . All of the  $\xi$ -dependent terms in Eq. (10) are collected in the function  $\Phi(\xi)$  so that the interference term (in square brackets) is generation-region independent. This reflects the successful liberation of entangled photon pairs from any influence of dispersion associated with the nonlinear crystal which results in the width of the quantum-interference pattern being solely determined by the pump-pulse duration  $\tau_p$ .

*Quantum-interference experiments with the new cascaded-transformation state.*—Experiments were carried out using the experimental arrangement shown in Fig. 1. An actively mode-locked Ti:sapphire laser (pumped by a cw Ar-ion laser) emitted pulses of light at 830 nm. This radiation was frequency doubled to provide 80 fsec pulses (FWHM) at  $\lambda_p = 415$  nm, with a repetition rate of 82 MHz and an average power of 15 mW.

This sequence of femtosecond pulses was delivered to a BBO crystal, where it underwent type-II spontaneous parametric down-conversion in a collinear degenerate ( $\omega_1^0 = \omega_2^0 = \omega_p^0/2$ ) configuration. The collinear beam of down-converted photons was selected by a 2.5 mm circular aperture located about 7 cm beyond the crystal. After the residual pump pulses were separated from the signal and idler beams using a fused-silica dispersion prism, the down-converted photons were sent through the first stage of the cascaded-transformation apparatus which was composed of a birefringent material (crystalline quartz) whose fast axis was oriented at  $45^\circ$  with respect to the  $(H, V)$  laboratory coordinate system. Relative optical delay was introduced by changing the thickness of the birefringent material. The down-converted photons continued to a nonpolarizing beam splitter. The light at each output port of the beam splitter was then directed toward

a Glan-Thompson polarization analyzer, and thence to a convex lens for focusing onto an actively quenched Peltier-cooled avalanche photodiode photon-counting detector. The detector event registrations were conveyed to a coincidence circuit with a 3 ns coincidence-time window. Corrections for accidental coincidences were not necessary.

*Discussion.*—The experimental data (solid squares) illustrated in the insets of Fig. 3 display the coincidence rates (CR) as a function of polarization-analyzer angle  $\theta_2$  when  $\theta_1$  is fixed at  $0^\circ$ . Coincidence patterns are shown for two representative relative optical-path delays:  $-96$  fsec (left inset) and  $0$  fsec (right inset). The visibility of the coincidence pattern,  $\mathcal{V} = (\text{CR}_{\text{max}} - \text{CR}_{\text{min}})/(\text{CR}_{\text{max}} + \text{CR}_{\text{min}})$ , at  $0$  fsec delay turns out to be 96%, which is just a hair short of the theoretical maximum of 100%. This is to be compared with the rather anemic visibility of 17% that was observed in a similar configuration using a non-rotated polarization delay line [9] (time-delay fringes and polarization-rotation fringes are interchangeable, as has recently been established for femtosecond down-conversion [11]).

Coincidence patterns such as the representative ones shown in the insets were collected for various values of the relative optical-path delay  $\tau$ . The open squares in the main plot of Fig. 3 represent the maximum and minimum values of the coincidence rate (connected by a vertical dashed line) for various values of  $\tau$ . The coincidence-rate visibility is seen to decrease as  $\tau$  deviates from  $0$  fsec. The width of the coincidence-rate pattern, which is about 80 fsec, is determined solely by the pump-pulse duration and demonstrates a crystal-dispersion-independent profile

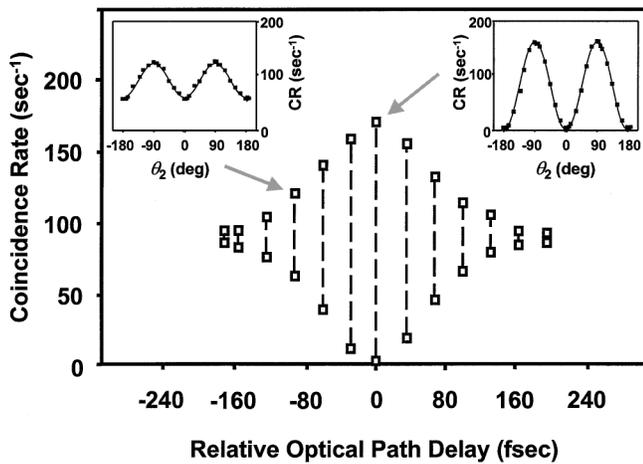


FIG. 3. Maxima and minima (open squares) of coincidence-rate quantum-interference patterns plotted as a function of relative optical-path delay time  $\tau$ . Each pair of data points (connected by a dashed vertical line) corresponds to the maximum and minimum coincidence rates (CR,  $\text{sec}^{-1}$ ) obtained from a polarization experiment at a particular value of  $\tau$  (examples are shown in the insets) in which the angle  $\theta_2$  of the second polarization analyzer is varied while  $\theta_1$  is fixed at  $0^\circ$ . The visibility of the interference pattern is measured to be 96% at 0 fsec relative path delay and the width of the path-delay interference pattern nicely reveals the 80 fsec pump-pulse duration used in these experiments. This experiment was conducted using a 3 mm thick BBO crystal.

that remains identical for different thicknesses of the non-linear down-conversion crystal.

To conclude, we have successfully co-opted the dual energy time and polarization entanglement inherent in type-II parametric down-conversion to implement a new scheme for eliminating the effects of decoherence in femtosecond parametric down-conversion. Despite our use of long non-linear crystals, which permits high photon-flux densities to be obtained, we have obtained a high-visibility quantum-interference pattern without the use of spectral filtering. We expect that this new technique will find use in a range of quantum-optics applications.

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*Note added in proof.*—Since completing this work, a recently published paper [12] describing an experiment that achieves a visibility of 64% using a different technique has come to our attention.

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