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# Two-photon anti-correlation in a Hanbury Brown–Twiss type experiment

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**Abstract**

A pair of orthogonally polarized signal and idler light quanta, degenerate in frequency and propagation direction, produced from type II parametric down conversion is sent through a Hanbury Brown–Twiss type experimental setup. We observed "anti-correlation" behavior of the pair in a coincidence photon counting measurement under certain experimental conditions.

In optical spontaneous parametric down conversion, a pair of light quanta is produced simultaneously. The pair is entangled [1,2], by means of the well known phase matching condition,

$$\omega_1 + \omega_2 = \omega_p, \qquad \boldsymbol{k}_1 + \boldsymbol{k}_2 = \boldsymbol{k}_p, \tag{1}$$

where $\omega$ ($\boldsymbol{k}$) represents the frequencies (wave number vectors) for signal (1), idler (2) and pump (p) of the down conversion. The quantum mechanical entanglement of the parametric down conversion light quanta pair has drawn a great deal of attention, since the first demonstration of it in an Einstein–Podolsky–Rosen–Bohm experiment [3]. Several experiments have observed two-photon "anti-correlation" in a Mach–Zehnder type interferometer by using type I parametric down conversion [4–6]. In these experiments, the nondegenerate (in propagation direction) photon pair is injected from two input ports of a beam splitter and detected coincidently by two photon counting detectors placed in the two output ports of the beamsplitter. The coincidence counting rate shows a minimum value when the pair takes equal path to reach the beamsplitter. Theoretical studies have been given to explain these experiments [7,8]. It is an observation of "anti-correlation" from the point of view of coincidence measurement, on the other hand it is also an observation of "photon bunching" in the sense that the pair always "bunched" together to reach the same detector. The above observation is an example of two-photon interference phenomenon.

We wish to report in this paper an experiment in which "anti-correlation" is observed in a Hanbury Brown–Twiss [9] type experimental setup using type II down conversion. A pair of orthogonally polarized signal and idler light quanta (one o-ray, another e-ray, of the down conversion crystal), generated from type II parametric down conversion, is injected collinearly through a single port of a beamsplitter and detected by two photon counting detectors in the two output ports of the beamsplitter. See Fig. 1. When the photon pair takes equal path to reach the beamsplitter, the coincidence counting rate is zero. However, the coincidence counting rate is off from the zero value when an optical delay between the o-ray and the e-ray light quanta is introduced by inserting

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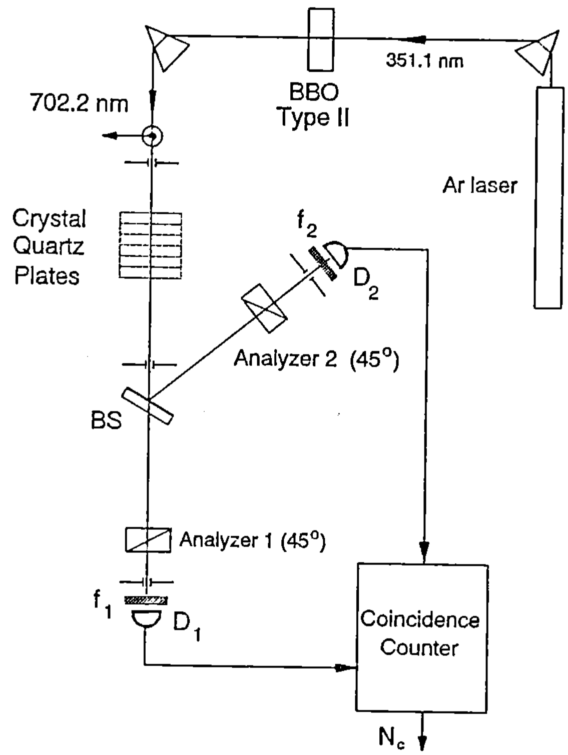



Fig. 1. Schematic experimental setup.

a set of crystal quartz plates in the beam path. The "anti-correlation" is completely removed when the optical delay is greater than the coherence length of the fields.

The schematic experimental setup is illustrated in Fig. 1. A cw argon ion laser line of 351.1 nm was used to pump an 8 mm×8 mm× (0.50±0.05) mm BBO ($\beta$-$BaB_2O_4$) nonlinear crystal. The BBO was cut at a type II phase matching angle to generate a pair of orthogonally polarized signal and idler photons collinearly at 702.2 nm wavelength. The down converted beams were separated from the pumping beam by a UV grade fused silica dispersion prism, then directed collinearly at a near normal incident angle to a polarization independent beamsplitter which has 50%–50% reflection and transmission coefficients. A single photon detector is placed in each transmission and reflection output port of the beamsplitter. The photon detectors are dry ice cooled avalanche photodiodes operated in photon counting Geiger mode. A Glan Thompson linear polarization analyzer, followed by a narrow bandwidth interference spectral filter, is placed in front of each of the detectors. The polarization analyzers are oriented at 45° relative to the o-ray and e-ray polarization planes of the BBO crystal. The spectral filters $f_1$ and $f_2$ have Gaussian shape transmission functions centered at 702.2 nm. Two different bandwidths of the spectral filters (in pair) are used in the experiment: a pair of 3.4 nm and a pair of 9.0 nm (full width at half maximum), respectively. The output pulses of the detectors are then set to a coincidence circuit with a 3 ns coincidence time window. The two detectors are separated by about 2 m.

A set of crystal quartz plates is placed between the down conversion crystal and the beamsplitter for changing the optical delay $\delta$ between the orthogonally polarized signal and idler photons, taking advantage of the anisotropic property of the refractive indices of the quartz plates. The fast axes of the quartz plates were carefully aligned to match the o-ray or e-ray polarization planes of the BBO crystal during the measurements. Each of the quartz plates is $1\pm0.1$ mm in thickness, resulting in an optical delay $\delta\simeq30$ fs between the o-ray and e-ray at wavelengths around 700 nm. The quartz plates were aligned carefully one by one before taking of data and moved away one by one during the measurements. Two sets of measurements were made in order to have a symmetrically distribution of the experimental points. We aligned the fast axes of the quartz plates to match the o-ray or the e-ray polarization plane of the BBO, respectively, in each of the two sets of measurements.

Fig. 2 reports typical observed "anti-correlation" coincidence rate measurements as a function of the optical delay $\delta$. The two curves correspond to the two different bandwidths of the spectral filters, 3.4 and 9.0 nm, respectively. The coincidence counts are direct measurements, with no "accidental" subtractions or any other theoretical corrections. Each of the data points corresponds to different numbers of quartz plates remaining in the path of the signal and idler beams. The left (right) side points, which are indicated by the minus sign (plus sign), of data were taken under the following condition: the fast axes of the quartz plates coincide with the o-ray (e-ray) polarization plane of the BBO crystal. The modulation: (maximum − minimum)/(maximum + minimum) is about 97 ± 2%.

Contrary to the coincidence counting rate, the single detector counting rate remains constant when $\delta$ is changed, as is reported in Fig. 3.

This phenomenon is a typical quantum two-photon interference effect, even though there is no interferometer involved. The interference "anti-correlation" is the result of the quantum superposition (cancellation) of the two-photon probability amplitudes,

$$|\text{o-ray transmitted to detector } 1\rangle \otimes |\text{e-ray reflected to detector } 2\rangle\,,$$

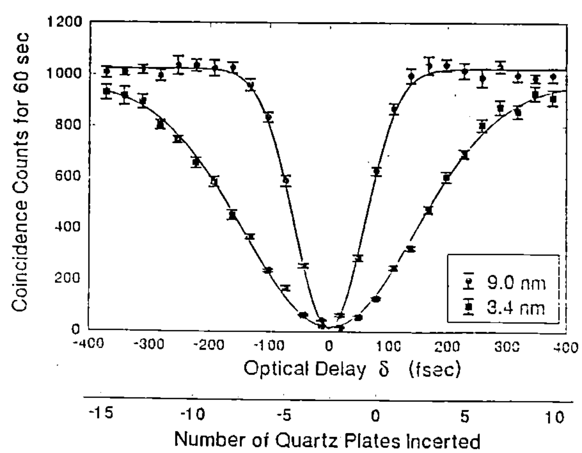
$$|\text{e-ray transmitted to detector } 1\rangle \otimes |\text{o-ray reflected to detector } 2\rangle\,, \tag{2}$$

when the orthogonally polarized photon pair is incident through a single port of the beamsplitter. To explain our experiment, we present a simple quantum mechanical model.

According to the standard theory of parametric down conversion, the two-photon state can be written as [10,11],

$$|\Psi\rangle = \int \mathrm{d}\omega_p\, A(\omega_p) \int \mathrm{d}\omega_1\, \mathrm{d}\omega_2\, \delta(\omega_1+\omega_2-\omega_p)\, a_o^\dagger(\omega_1)\, a_e^\dagger(\omega_2)\,|0\rangle\,, \tag{3}$$

where $\omega$ represents the frequencies for signal (1), idler (2) and pump (p) of the down conversion. The delta function represents perfect frequency phase matching of the down conversion, i.e., $\omega_1+\omega_2=\omega_p$. The wave number phase matching condition $\boldsymbol{k}_1+\boldsymbol{k}_2=\boldsymbol{k}_p$ is implicit in the choice of the locations of the pinholes which direct the down converted beams to the detectors; in this experiment we consider collinear down conversion. The



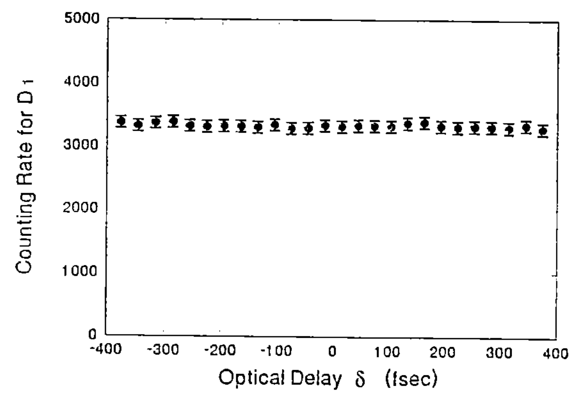




Fig. 2. Coincidence counts in 60 s as a function of the optical delay, $\delta$, between signal and idler photons, which corresponds to a certain number of quartz plates. The solid curves are fitting curves of Eq. (9). The bandwidths are 3.4 and 9.0 nm, respectively.

Fig. 3. Single detector counting rate remains constant when $\delta$ is changed.

subscripts o and e for the creation operators indicate the ordinary and extraordinary rays of the down conversion, traveling along the same direction as the pump, the $z$-direction. The defined $x$ and $y$ coordinate axes coincide with the o-ray and the e-ray polarization directions of the crystal. $A(\omega_p)$ is a spectral distribution function for the laser line, which is usually considered to be a Gaussian.

The fields at the detectors 1 and 2 are given by

$$E_1^{(+)}(t)=\alpha_t \int d\omega f(\omega)[\exp(-i\omega t_1^o)\hat{e}_1\cdot\hat{e}_o a_o(\omega)+\exp(-i\omega t_1^e)\hat{e}_1\cdot\hat{e}_e a_e(\omega)],$$

$$E_2^{(+)}(t)=\alpha_r \int d\omega f(\omega)[\exp(-i\omega t_2^o)\hat{e}_2\cdot\hat{e}_o a_o(\omega)+\exp(-i\omega t_2^e)\hat{e}_2\cdot\hat{e}_e a_e(\omega)], \tag{4}$$

where $\hat{e}_i$ is in the direction of the $i$th linear polarization analyzer axis, $a_0(\omega)$ and $a_e(\omega)$ are the destruction operators for the o-ray and the e-ray, $\alpha_t$ and $\alpha_r$ are the complex transmission and reflection coefficients of the beamsplitter, and $f(\omega)$ is the spectral transmission function of the filters. The $t_i$ are given by: $t_i^o=t-l_i^o/c$, $t_i^e=t-l_i^e/c$, $i=1, 2$, where $l_i^{o,e}=\int dz\, n^{o,e}(z)$, indicates the optical path for o-ray or e-ray of the $i$th beam, with $n^{o,e}(z)$ being the refractive index at position $z$, we have approximated $(dn/d\omega)_0-(dn/d\omega)_e\simeq 0$ for simplifying the calculation. The use of pinholes, which limit the transverse width of the beams, allows a good one-dimension approximation. The detectors will be treated as point detectors to be located at $z_1$ and $z_2$. The coincidence counting rate is given by

$$R_c=\frac{1}{T}\int_0^T\int dT_1\, dT_2\, \langle\Psi|E_1^{(-)}E_2^{(-)}E_2^{(+)}E_1^{(+)}|\Psi\rangle S(\tau,\Delta T_c)$$

$$=\frac{1}{T}\int_0^T\int dT_1\, dT_2\, |\Psi(t_1,t_2)|^2 S(\tau,\Delta T_c), \tag{5}$$

where $\tau\equiv T_1-T_2$, $T_i$ is the detection time of the $i$th detector, $S(\tau, \Delta T_c)$ is a function that describes the coincidence circuit, and $\Delta T_c$ is the time window of the coincidence circuit. For $\tau>\Delta T$, $S(\tau, \Delta T_c)\simeq 0$, and for $\tau<\Delta T$, $S(\tau, \Delta T_c)\simeq 1$. If the coincidence time window is large enough, $S$ can be considered as one at time $t$. The time integral (data collection time $T$) can be taken to infinity as a good approximation. In Eq. (5), an effective two-photon wavefunction $\Psi(t_1, t_2)$, which is realized by the coincidence measurement at the two detectors, is defined by

$$\Psi(t_1,t_2)=\langle 0|E_1^{(+)}(t_1)E_2^{(+)}(t_2)|\Psi\rangle. \tag{6}$$

The introduction of $\Psi(t_1, t_2)$ is helpful for the understanding of physics.

It is straightforward to show from (3)–(6),

$$\Psi(t_1,t_2)=\alpha_t\alpha_r[\hat{e}_1\cdot\hat{e}_o\hat{e}_2\cdot\hat{e}_e A(t_1^o,t_2^e)-\hat{e}_1\cdot\hat{e}_e\hat{e}_2\cdot\hat{e}_o A(t_1^e,t_2^o)], \tag{7}$$

where $A(t_1, t_2)$ is calculated in the Appendix. The two terms in (7) correspond to the two-photon probability amplitudes in (2). A phase shift of $\pi$ due to reflection has been taken into account. It is clear to see that if there is no optical delay between the o-ray and e-ray, the two terms in (7) will cancel each other for 45° oriented polarizers.

For Gaussian filters $f(\omega)$ with bandwidth $\sigma$ and a Gaussian spectral distribution of the pump field with bandwidth $\sigma_p$,

$$A(t_1,t_2)=A_0\exp[-\tfrac{1}{8}\sigma_p^2(t_1+t_2)^2]\exp[-\tfrac{1}{4}\sigma^2(t_1-t_2)^2]\exp(-i\Omega_1 t_1)\exp(-i\Omega_2 t_2), \tag{8}$$

where $\Omega_i$ is the $i$th filter's center frequency which is related to the peak frequency of the pump, $\Omega_p$, by $\Omega_1+\Omega_2=\Omega_p$ (in this experiment $\Omega_1=\Omega_2$). For the single line pump, $\exp[-\frac{1}{8}\sigma_p^2(t_1+t_2)^2]$ can be taken equal to one for a good approximation. Substituting (7) into (5), the coincidence counting rate is calculated as,

$$R_c = R_{c0}[1-\exp(-\tfrac{1}{2}\sigma^2\delta^2)]\,, \tag{9}$$

where $\delta \equiv \Delta l/c$ is the optical delay between the o-ray and the e-ray. The 45° angles between the analyzers and the o-ray polarization direction of the BBO have been taken into account. We use a right-handed natural coordinate system with respect to the $\boldsymbol{k}$ vector as the positive $z$ axis direction. Care has to be taken to follow the rules of the natural coordinate system, especially for the reflected beam. Note that the positive direction of the $x$ axis for the reflected beam is opposite to that of the transmitted beam.

Eq. (9) indicates an interference cancellation, i.e., "anti-correlation" at $\delta=0$. The "anti-correlation" vanishes exponentially when $\delta$ is off from zero. The solid curves in Fig. 2 are fitting curves of Eq. (9) for 3.4 and 9.0 nm bandwidths (full width at half maximum) of the spectral filters. The theory curves agree with the data within reasonable experimental error.

It is interesting to see that the zero point of the counting rate is not in the case of zero quartz plates. The reason is that the refractive indices for the o-ray and e-ray of the BBO crystal also contribute to the optical delay $\delta$, $\delta = \delta_{\mathrm{BBO}} + \delta_{\mathrm{quartz}}$. BBO is a negative uniaxial crystal and quartz is a positive uniaxial crystal. If we align the BBO and the quartz plates in such way that the o-ray and e-ray polarization planes coincide, the optical delay inside the BBO can be compensated by the quartz plate for a certain value of thickness. From repeated measurements, we conclude that the zero point of the coincidence counting rate happens at a point in which about $2.4\times(1\pm0.1)$ mm quartz plates are placed in the beam path. The difference of refractive indices $(n_e - n_o)$ of quartz is about 0.009 around 700 nm. $c/u_o - c/u_e \simeq n_o - n_e$ of BBO at this phase matching angle for 702.2 nm is about 0.077 [#1], where $u_{o,e}$ are the group velocities for the o-ray and e-ray, respectively. It indicates that 2.4 mm quartz plate will compensate for about half of the thickness of the BBO crystal which was used in the measurements. This effect suggests that the average "birth place" of the twin brother photon pair is in the middle of the down conversion crystal. This is a very reasonable suggestion. The down conversion efficiency is very low, so that the power of the pump beam can be treated as a constant and the intensity of the down converted beam grows linearly with the length of the crystal. The probability of creating a pair of light quanta inside the crystal is consequently a constant along the path of the pump.

In summary: In a Hanbury Brown–Twiss like experimental setup, we have demonstrated "anti-correlation" behavior of a photon pair with $97\pm2\%$ modulation. Under certain experimental conditions, the pair never separated to trigger detector 1 and detector 2, respectively, at the same time. This phenomenon is a typical quantum mechanical two-photon interference phenomenon. The interference is due to the superposition (cancellation) of the two-photon quantum mechanical probability amplitudes in (2). The "anti-correlation" vanishes when the optical delay between the o-ray and e-ray is greater than the coherence length of the down converted fields, resulting in a nonoverlap of the two-photon wave packets.

The above experiment has certain applications for precise measurement of refractive index dispersion, time delay and related physical quantities.

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## Appendix

From (3), (4) and (6),

$$A(t_1,t_2) = \int d\omega_p\, A(\omega_p) \int\!\!\int d\omega_1\, d\omega_2\, f_1(\omega_1) f_2(\omega_2)\,\delta(\omega_1+\omega_2-\omega_p)\exp(-i\omega_1 t_1)\exp(-i\omega_2 t_2)\,, \tag{A.1}$$

[#1] The refractive indices of BBO were provided by the Fujian Institute of Research on Structure of Matter, China.

where $t_j = t - l_j/c$, $j=1, 2$, $l_j = \int \mathrm{d}z\, n(z)$, $n$ is the refractive index, dispersion is not taken into account for simplifying the calculation. Assuming Gaussian spectrum distributions with equal widths $\sigma$ for both signal and idler,

$$f_j(\omega_j) = f_0 \exp[-(\omega_j - \Omega_j)^2/2\sigma^2], \qquad j=1,2\,. \tag{A.2}$$

Substituting into (A.1) and making the change of variables $\nu_+ = \nu_1 + \nu_2$, and $\nu_- = \frac{1}{2}(\nu_1 - \nu_2)$ gives

$$\begin{aligned} A(t_1, t_2) = &\int \mathrm{d}\omega_\mathrm{p}\, A(\omega_\mathrm{p}) \exp[-\tfrac{1}{2}\mathrm{i}\Omega_\mathrm{p}(t_1+t_2)] \exp[-\tfrac{1}{2}\mathrm{i}\Omega_\mathrm{d}(t_1-t_2)] \\ &\times \int \mathrm{d}\nu_+ \int \mathrm{d}\nu_- f_0^2 \exp(-\nu_+^2/4\sigma^2) \exp(-\nu_-^2/\sigma^2) \\ &\times \exp[-\tfrac{1}{2}\mathrm{i}\nu_+(t_1+t_2)] \exp[-\mathrm{i}\nu_-(t_1-t_2)]\delta(\nu_+ - \Delta\omega_\mathrm{p})\,, \end{aligned} \tag{A.3}$$

where $\Delta\omega_\mathrm{p} = \omega_\mathrm{p} - \Omega_\mathrm{p}$, and $\Omega_\mathrm{d} = \frac{1}{2}(\Omega_1 - \Omega_2)$. The integral (A.3) over $\nu_+$ is easily done using the delta function. In doing the integral over $\nu_-$ we use the fact that $\Omega_1$ and $\Omega_2$ are both much greater than $\sigma$ to extend the integral from $-\infty$ to $\infty$. This is just a Gaussian integral which leads to the result,

$$\begin{aligned} &A(t_1, t_2) = u(t_1 - t_2)v(t_1 + t_2), \qquad u(t) = \exp(-\tfrac{1}{4}\sigma^2 t^2) \exp(-\tfrac{1}{2}\mathrm{i}\Omega_\mathrm{d} t)\,, \\ &v(t) = \exp(-\tfrac{1}{2}\mathrm{i}\Omega_\mathrm{p} t) \int \mathrm{d}\omega_\mathrm{p}\, A(\omega_\mathrm{p}) \exp(-\tfrac{1}{2}\mathrm{i}\Delta\omega_\mathrm{p} t) \exp(-\Delta\omega_\mathrm{p}^2/4\sigma^2)\,, \end{aligned} \tag{A.4}$$

where all constants have been incorporated into $A(\omega_\mathrm{p})$. For a Gaussian spectrum and narrow bandwidth of $A(\omega_\mathrm{p})$ $(\Delta\omega_\mathrm{p} \ll \sigma)$, (A.4) can be approximated,

$$A(t_1, t_2) = A_0 \exp[-\tfrac{1}{8}\sigma_\mathrm{p}^2(t_1+t_2)^2] \exp[-\tfrac{1}{4}\sigma^2(t_1-t_2)^2] \exp(-\tfrac{1}{2}\mathrm{i}\Omega_1 t_1) \exp(-\tfrac{1}{2}\mathrm{i}\Omega_2 t_2)\,. \tag{A.5}$$

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