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Tunable Bell-inequality violations by non-maximally-violating states in type-II parametric down-conversion

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We use a tunable quantum interference in type-II parametric down-conversion to construct two-photon quantum states that exhibit less-than-maximal, and tunable, violations of Bell-type inequalities. These states, no longer singlet-state analogs, possess an interference term of controllable magnitude that we adjust by tuning the measurement bandwidth. We show violations of two Bell-type inequalities in polarization variables, one more general than the other. We use these tunable violations to probe the threshold degree of interference that must be present to generate a violation, using an appropriate quantitative figure of merit.

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I. INTRODUCTION

In previous work (atomic cascade [1–3], type-I [4,5], and type-II [6] experiments) with optical photon violations of Bell-type inequalities in polarization variables, the quantum state of interest has been a photon polarization analog of the spin-$\frac{1}{2}$ singlet state. Such analogs provide the maximum theoretical quantum violation for two particles of a Bell-type inequality. The experimental tests [1–6] have confirmed this, up to modest experimental nonidealities and detection efficiency losses.

In this work we generate states that are no longer singlet-state analogs. They lack the rotational symmetry of singlet-state analogs, and exhibit weaker quantum interference. We show with data that we can tune both the interference and the degree to which two different Bell-type inequalities in polarization variables are violated.

We use the quantum state of orthogonally polarized type-II pairs generated in parametric down-conversion to exhibit these Bell-inequality violations. The state exhibits a bandwidth-dependent interference that lessens as the measurement bandwidth grows. As shown in Ref. [7], the strength of the interference term of interest can be tuned over a wide range.

Specifically, we show Bell-inequality violations of greater than three standard deviations for quantum states characterized by values of $\rho=0.84$, 1.53, and 1.73. Here ρ is a dimensionless parameter (equal to 2 for a singlet-state analog) useful [7] in quantifying the magnitude of the tunable interference. For values of ρ less than 2, the quantum state lacks the rotational symmetry of a single-state analog, and generates weaker violations of Bell-type inequalities. We report such violations, using the tunable feature to probe the threshold value of ρ below which Bell-type inequalities cease to be violated. We shall show that such a threshold depends, in part, on the particular Bell-type inequality under consideration.

The nonsinglet states measured here differ from other [8–11] constructions of less-than-maximally violating states. In those constructs, the coefficients of the terms in the multiterm states are adjusted to give unequal weighting on each term. Here, each term of the two-term state is weighted equally, and the wash-out of the relative phase between them is controlled by ρ. We emphasize that the present work is an experimental relaxation of generating nonmaximal entanglement [12], in a tunable fashion, and in a manner different from the theoretical approaches proposed already [8–11].

II. STATES OF REDUCED INTERFERENCE

Our experimental setup is described in Ref. [7], where a coincidence detection measurement using linear polarization analyzers is made on the two-photon quantum state produced in type-II down conversion and incident on a nonpolarizing beam splitter. The coincidence behaviors are well summarized by a probability

$$P_{12}\propto\sin^2\theta_1\cos^2\theta_2+\cos^2\theta_1\sin^2\theta_2 -\rho\,\sin\theta_1\sin\theta_2\cos\theta_1\cos\theta_2, \qquad (1)$$

with analyzers in front of the two detectors ($D1$ and $D2$) set at angles θ_1 and θ_2, respectively. Here ρ is a coefficient that we have found to depend on two variables: the bandpass of the filters used in front of the detectors, and the BBO crystal length.

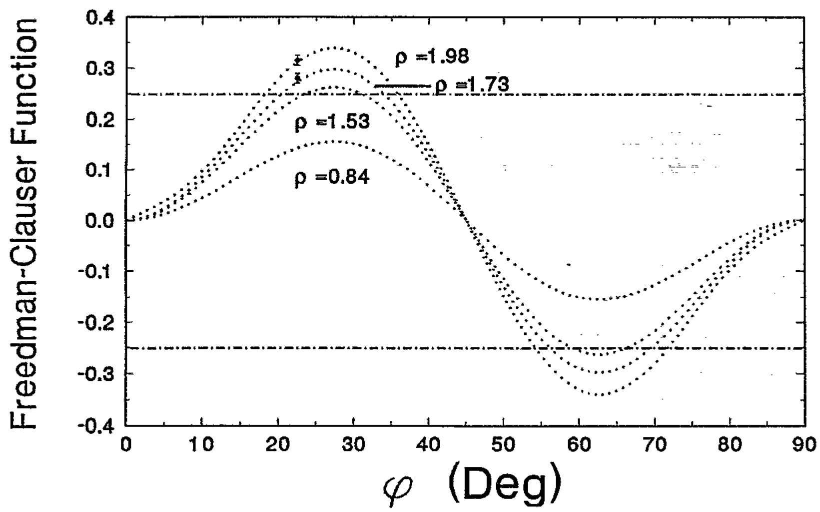

FIG. 1. Violations of the Freedman form for $\rho<2$. The left-hand side of Eq. (2) as a function of ϕ plotted for $\rho=1.98$, 1.73, 1.53, and 0.84. The curves are generated using the theoretical prediction of Eq. (3) multiplied by the experimentally measured values of η_1, η_2. Measured violations for the first two values are shown.

With the longer crystal, and for bandpasses greater than 1 nm with the shorter crystal, we measured ρ to be less than the ideal value of 2 [7]. For any value of ρ, Eq. (1) cannot be factored into a function of θ_1 only, multiplied by a function of θ_2 only. This nonfactorizability is due to the entangled [12] nature of the quantum state. It is difficult to generate the θ dependence of the ρ-dependent term by a classical mechanism, but quantum mechanically it arises from a cross term generated when two probability amplitudes, one representative of "ordinary (o) ray sent to $D1$, extraordinary (e) ray sent to $D2$" and one representative of "e ray sent to $D1$, o ray sent to $D2$," are added coherently. The two-photon quantum state at our beam splitter has these two amplitudes, which must have a definite phase relation between them to produce the ρ-dependent interference term.

One can ask of what value are the states of reduced interference with ρ less than the ideal value of 2. The application here is in answer to this question. The interference, though not ideal, is nevertheless of sufficient magnitude to enable the coincidence counts to violate Bell-type inequalities. All such violations disprove Bell's two postulates [13,14] of locality and reality to hold in describing the behavior of quantum particles. These two postulates are true for nature as described by classical physics, and have intuitive appeal. Therefore, violations by quantum particles cast in quantitative terms the types of behavior allowed in quantum mechanics but not in any local reality theory.

III. TUNABLE BELL-INEQUALITY VIOLATIONS

The quantum states generating the $\rho<2$ behavior are predicted to violate Bell-type inequalities to a lesser degree than the singlet-state analog achieved with $\rho=2$. We achieve this comparison in two ways. The first is to show violations of the Freedman-Clauser form of the Bell inequality. Because of limitations of this method, we then present a more general Bell inequality, motivate an appropriate figure of merit to use in quantifying the strength of an experimental violation, and show violations with our measurements.

We first consider the Bell inequality form of Freedman and Clauser [14,15],

$$\left|\frac{N_{12}(\phi)-N_{12}(3\phi)}{N_{12}(-,-)}\right| \leqslant 0.25, \tag{2}$$

for $\phi \equiv \theta_1-\theta_2=22.5°$, $N_{12}(\phi)$ representing coincidence counts collected in some length of time, and $N_{12}(-,-)$ representing coincidence counts in the same length of time with both analyzers removed. This form is applicable here for all values of ρ, because all but one of the required symmetry properties of the system used in its derivation hold irrespective of the value of ρ. These symmetry properties are that coincidence counts be independent of the analyzer angle when the other analyzer is removed, and that singles in each detector be independent of the analyzer angle in front of the detector.

The one requisite symmetry property that is broken for $\rho<2$ is a "rotational invariance" that $P_{12}(\theta_1,\theta_2)$ depend on ϕ only. The coincidence counts of Eq. (1) are no longer a function of only one variable, as in the situation with $\rho=2$. To show this, we rewrite Eq. (1) in the form

$$P_{12} \propto \left(\frac{2+\rho}{4}\right)\sin^2(\theta_1-\theta_2)+\left(\frac{2-\rho}{4}\right)\sin^2(\theta_1+\theta_2), \tag{3}$$

showing coincidences for $\rho<2$ to depend not only on the variable $\phi \equiv \theta_1-\theta_2$, but also on the variable $\Sigma \equiv \theta_1+\theta_2$. The specification of ϕ leaves one more degree of freedom, that of the sum angle (Σ), unknown. This breaks rotational invariance, forcing us to examine the consequences for derivations of Bell-type inequalities. An examination of the derivation of Freedman and Clauser shows it to be upheld if we impose a constraining condition that this second variable

TABLE I. Bell-inequality angles for $\rho<2$.

ρ	θ_1 (deg)	θ_1' (deg)	θ_2 (deg)	θ_2' (deg)
1.98 ± 0.04	22.5	67.5	135	90
1.73 ± 0.04	22.5	67.5	135	90
1.53 ± 0.04	11.25	60	120	82.5
0.84 ± 0.03	0	45	105	75

TABLE II. Bell-inequality measurements: coincidence counts for $\rho<2$.

ρ	$N(\theta_1',\theta_2')$	$N(\theta_1',\theta_2)$	$N(\theta_1,\theta_2)$	$N(\theta_1,\theta_2')$	$N(\theta_1,-)$	$N(-,\theta_2)$
1.98 ± 0.04	951	4060	3701	4054	4534	5060
1.73 ± 0.04	2021	8302	8040	9044	10 213	10 790
1.53 ± 0.04	5138	17 150	20 846	22 040	24 860	26 626
0.84 ± 0.03	8936	13 477	20 829	20 981	22 373	24 479

(Σ) equal 3ϕ; i.e., assume the value 67.5° (mod π) when $\phi=22.5°$, and 22.5° (mod π) when $\phi=67.5°$.

We took measurements under these conditions for the (3.1 nm, 3.1 nm) filter combination generating $\rho=1.73\pm0.03$. The result for the left-hand side of Eq. (2) is 0.2798 ± 0.0045, less than the $\rho=1.98\pm0.04$ result [6] of 0.316 ± 0.003. These two measurements are plotted in Fig. 1. The $\rho=1.73$ quantum state, though not a singlet-state analog, is nevertheless able in coincidence counts to violate the Freedman-Clauser form of the Bell inequality.

By comparison, the quantum prediction for (2), for a singlet-state analog and 100% efficient analyzers, is $0.25\sqrt{2}\approx0.354$. Our $\rho=2$ value of $0.316\pm0.003(1\sigma)$ is less than the perfect quantum prediction because of passive, polarization-independent losses at the analyzer faces, which were not optimally coated for the 702-nm wavelength. These losses were measured as efficiencies η_1,η_2 of analyzers 1 and 2 (0.905 ± 0.014 and 0.976 ± 0.015, respectively), and imply a quantum-mechanical violation of (2) by a $\rho=2$ state of 0.312 ± 0.018.

Under the condition that the sum angle be always equal to 3ϕ, the magnitude of the left-hand side of Eq. (2) is predicted to be less than its value for $\rho=2$ by the factor $\rho/2$. These predictions are graphed in Fig. 1 for the four values of ρ (1.98, 1.73, 1.53, and 0.84) that we measured. This figure shows that the states with $\rho<2$ are still possible to generate from Eq. (2) violations of the bound (0.25) imposed by Bell's two postulates.

The problem we have addressed so far is the specification of the sum angle. Because Eq. (2) only represents a valid Bell-inequality expression for certain choices of the sum angle, these choices are a constraining feature. This constraint leads to the aforementioned suppression factor of $\rho/2$ which, as Fig. 1 shows, prevents values of ρ less than a threshold value (~1.58 using the measured values for η_1,η_2) from generating violations. With ideal analyzers (η_1,η_2 both equal to 1), the threshold value of ρ is $\sqrt{2}$.

The quantum states for values of ρ below this threshold can nonetheless exhibit violations of other, more general Bell-type inequalities, specifically variations of the Clauser-Horne-Shimony [14] form. The first is

$$[-N_{12}(\theta_1',\theta_2')+N_{12}(\theta_1',\theta_2)+N_{12}(\theta_1,\theta_2')+N_{12}(\theta_1,\theta_2)] -[N_{12}(\theta_1,-)+N_{12}(-,\theta_2)]<0, \tag{4}$$

in which the Clauser-Horne no-enhancement assumption [16] has already been imposed, and in which probabilities have been converted to coincidence counts N_{12} accumulated in some time interval, as before.

Although we have generated violations of (4), we advocate a stronger version in which the transmission losses of the analyzers are recognized and removed. The basis for this is a generalized version of the no-enhancement hypothesis, in which the passive, polarization-independent analyzer losses are assumed not to affect the behavior of the source whose coincidence properties are under study. We note that these analyzer losses must be controlled [16,14,17] in a rigorous bell-inequality test. For our purpose here of exhibiting the coincidence behavior of the source, we use this generalization to alter (4) to the form

$$[-N_{12}(\theta_1',\theta_2')+N_{12}(\theta_1',\theta_2)+N_{12}(\theta_1,\theta_2')+N_{12}(\theta_1,\theta_2)] -[\eta_2N_{12}(\theta_1,-)+\eta_1N_{12}(-,\theta_2)]<0. \tag{5}$$

TABLE III. Bell-inequality violations for $\rho<2$ using counts of Table II.

ρ	Eq. (4)	Eq. (5)	$\eta_2N(\theta_1,-)$	$\eta_1N(-,\theta_2)$	$Q-1$	$Q_{\text{pred}}-1$
1.98 ±0.04	1188 ±143 (8σ)	1778 ±178 (10σ)	4425 ±97	4579 ±98	0.198 ±0.022	0.207 ±0.010
1.73 ±0.04	2362 ±211 (11σ)	3632 ±310 (11σ)	9968 ±188	9765 ±187	0.184 ±0.015	0.159 ±0.008
1.53 ±0.04	3412 ±326 (10σ)	6538 ±640 (10σ)	24 263 ±418	24 097 ±413	0.135 ±0.015	0.128 ±0.006
0.84 ±0.03	-501 ±333 no violation	2169 ±596 (3.6σ)	21 836 ±379	22 154 ±382	0.049 ±0.014	0.039 ±0.005

To violate these inequalities, we searched numerically for the four analyzer angle settings $(\theta_1, \theta_1', \theta_2, \theta_2')$ that would generate the maximal violation of these two equations. The angle choices we used, shown in Table I, are not a unique choice to achieve the same violation. The measurements at these angles are shown in Table II, and the calculated violations are shown in Table III.

We note that the Bell-inequality violation is a straightforward application of determining whether the coincidence count expression on the left-hand side of Eq. (4) or Eq. (5) is greater than 0. No symmetry properties about the system need be demonstrated. Therefore, the six coincidence measurements of these equations are the only measurements needed.

To assess the degree to which a violation has been shown, we advocate as an appropriate figure of merit the quantity $Q-1$, for Q the ratio of the quantity in Eq. (5) in square brackets to the quantity in parentheses. A violation generates a positive $(Q-1)$. The measured values of $Q-1$ are listed in Table III.

This figure of merit $Q-1$ is also useful to apply to Eq. (2). As derived by Freedman and co-workers [14,15], two Bell-inequality expressions, each with $Q-1=(\sqrt{2}-1)/2\sim 0.207$, are combined to yield the form (2), in which the quantum prediction exceeds the Bell bound by twice this factor, or $\sqrt{2}-1\sim 0.414$. That is, two $\sim 20\%$ violations were combined to yield the $\sim 40\%$ violation of Eq. (2).

The maximum value of $Q-1$ for Eq. (5) is attained for $\rho=2$, when $Q-1=(\sqrt{2}-1)/2\sim 0.207$. Violations (i.e., positive $Q-1$) are observed for ρ as low as 0.84 (see Table III), and are theoretically possible down to $\rho\approx 0.78$. The choices of angles generating the optimum $Q-1$ are for sufficiently small ρ no longer the same as for larger values of ρ. This confirms the understanding that the violations of Eq. (2) mentioned above, in which constraints are placed on the sum angle, do not, in general, represent the largest possible violations for a particular ρ.

From Table III, our measured violation of Eq. (5), a variation of the Clauser-Horne-Shimony version of the Bell inequality, is as large as 11σ. Our purpose here is not to measure a Bell-inequality violation to great precision, but rather to document the quantum nature of the two-photon state that is in view of the detectors.

To summarize the results of Table III, we observe violations of Bell-type inequalities with nonsinglet states, states with reduced interference (i.e., $\rho<2$) as compared with the singlet-state analog (for which $\rho=2$). These violations can be quantified with the figure of merit $Q-1$ as being weaker violations than those of singlet states.

[1] W. Perrie, A. J. Duncan, H. J. Beyer, and H. Kleinpoppen, Phys. Rev. Lett. **54**, 1790 (1985).
[2] A. Aspect, J. Dalibard, and G. Roger, Phys. Rev. Lett. **49**, 1804 (1982), and references therein.
[3] For a review of earlier work, see J. F. Clauser and A. Shimony, Rep. Prog. Phys. **41**, 1881 (1978).
[4] C. O. Alley and Y. H. Shih, in *Proceedings of the Second International Symposium on Foundations of Quantum Mechanics in the Light of New Technology, Tokyo, 1986,* edited by M. Namiki *et al.* (Physical Society of Japan, Tokyo, 1987), p. 47; Y. H. Shih and C. O. Alley, Phys. Rev. Lett. **61**, 2921 (1988).
[5] Z. Y. Ou and L. Mandel, Phys. Rev. Lett. **61**, 50 (1988).
[6] T. E. Kiess, Y. H. Shih, A.V. Sergienko, M. H. Rubin, and C. O. Alley, Phys. Rev. Lett. **71**, 3893 (1993).
[7] Y. H. Shih, A. V. Sergienko, M. H. Rubin, T. E. Kiess, and C. O. Alley, Phys. Rev. A **50**, 23 (1994).
[8] P. H. Eberhard, Phys. Rev. A **47**, R747 (1994).
[9] L. Hardy, Phys. Rev. Lett. **71**, 1665 (1993).
[10] S. Goldstein, Phys. Rev. Lett. **72**, 1951 (1994).
[11] P. G. Kwiat, P. H. Eberhard, A. M. Steinberg, and R. Y. Chiao, Phys. Rev. A **49**, 3209 (1994).
[12] M. A. Horne, A. Shimony, and A. Zeilinger, Phys. Rev. Lett. **62**, 2209 (1989).
[13] J. S. Bell, Physics **1**, 195 (1965); Rev. Mod. Phys. **38**, 447 (1966).
[14] For a review, see J. F. Clauser and A. Shimony, Rep. Prog. Phys. **41**, 1881 (1978).
[15] S. J. Freedman and J. F. Clauser, Phys. Rev. Lett. **28**, 938 (1972).
[16] J. F. Clauser and M. A. Horne, Phys. Rev. D **10**, 526 (1974).
[17] For recent discussions, see V. L. Lepore and F. Selleri, Found. Phys. Lett. **3**, 203 (1990).