Does Firm Heterogeneity Lead to Differences in Relative Executive Compensation?

Ana Albuquerque*
School of Management
Boston University
November 24, 2009

*This paper is based on the third chapter of my thesis dissertation entitled “Essays in Relative Performance Evaluation” at the University of Rochester. I am grateful to Leslie Marx for many detailed comments and suggestions. I also thank the referee, Rui Albuquerque, Alison Kirby Jones, Evgeny Lyandres, Claudine Mangen, Michael Raith for helpful comments. Address: Boston University, School of Management, 595 Commonwealth Ave., Boston, MA 02215, USA. Email: albuquea@bu.edu.
Abstract

Cost heterogeneity is an important source of performance disparity among firms. This heterogeneity conditions the strategic decisions that firms make in the product market and can lead to heterogeneity in the design of managerial compensation contracts. I investigate the effect of cost heterogeneity in a strategic product market environment where firms compete à la Cournot. The paper offers new predictions on how executive compensation contracts that account for relative performance must be adjusted for cost differences.
1 Introduction

Firms are intrinsically heterogeneous, even within the same industry. For example, firms produce differentiated products, face different demand curves, or differ in their efficiency. Heterogeneity conditions the strategic decisions that firms make and, not surprisingly, leads to heterogeneity in firm profitability. Because managerial compensation is often tied to measures of firm profitability, it is important to understand the effect that cost heterogeneity has on managerial compensation contracts.

I investigate the design of managerial compensation contracts when firms compete à la Cournot in the product market and managerial compensation is allowed to depend on own and rival firm’s profits. When firms are identical, the optimal level of compensation decreases in the rival firm’s profit: shareholders want their managers to be more aggressive in setting quantities as this strategy yields the highest profit for each firm. Moreover, this effect is more pronounced as products become more substitutable (Aggarwal and Samwick, 1999). When costs differ across firms, I show that as products become more substitutable, only shareholders of less efficient firms give incentives to harden product market competition. As products become more substitutable, shareholders of more efficient firms give their managers incentives to soften product competition, provided the relative difference in efficiency levels is large enough.

The result that cost-heterogeneous firms incentivize their managers differently has implications for the use of relative performance evaluation (RPE) in executive compensation. One of the main puzzles in the executive compensation literature has been the limited evidence of RPE in compensation schemes (e.g., Murphy, 1999). In the presence of cost heterogeneity, my results suggest that cross sectional analyses that fail to take into account the differential behavior of more efficient firms within

---

an industry introduce a bias regarding the relevance of RPE. The results supporting RPE usage when using industry-size peers may be due to a correlation between peer performance and the omitted variable causing the bias referred to above.

Ideally, to correct this bias one needs empirical measures of marginal costs to model relative performance usage. Alternatively, assuming that firms with lower marginal costs are also larger firms, then firm size can be used as a proxy for a firm’s efficiency level. In that case, the model provides a rationale for the empirical results in Albuquerque (2009), who finds evidence consistent with RPE when using peer firms in the same industry and size quartile, but no evidence of RPE when peer firms are those in the same industry irrespective of size.

This paper contributes to a large literature that explores the impact of firm heterogeneity on the design of executive compensation contracts. For example, Hermalin and Wallace (2001) analyze the savings and loan industry and find – even within a single industry – evidence of heterogeneity in the structure of compensation contracts. My results add to this literature by pointing to cost heterogeneity as a rationale for equilibrium heterogeneity in compensation contracts.

2 The Model

Consider the shareholders’ problem in designing the optimal compensation contract of a manager in a monopolistic competitive industry with two firms labelled 1 and 2. I also use the indexes i and j for the firms. Firms act strategically à la Cournot.\footnote{The results for Bertrand competition are similar and available upon request.} The model extends Fumas (1992) and Aggarwal and Samwick (1999) to allow for heterogeneity in marginal costs, $c_i$, with $c_1 \neq c_2$. The model with cost differences can be interpreted as a model where firm heterogeneity arises from demand differences as in Singh and Vives (1984). Accordingly, high-demand firms resemble low marginal
cost firms. For brevity, I present the results only with cost heterogeneity.

Firm i’s inverse demand function is

\[ P_i (q_i, q_j) = A - bq_i - aq_j, \]  

where \( q_i \) and \( q_j \) are quantities, and \( P_i \) is the price of good \( i \). The parameters \( A, b \) and \( a \) are all positive.

Firms in the industry face a common, independently and identically distributed profit shock, \( \varepsilon \), with zero mean. This shock invalidates contracts written on prices or quantities which are assumed to be unobservable. Thus, contracts can only be written as a function of profits. All agents are risk neutral. Managers face a participation constraint in that they have an outside job opportunity that gives them a payoff of \( w \).

The timing of the model is as follows. First, shareholders of each firm choose the contract’s parameters. Second, given shareholders’ actions, managers choose prices.

Consider a linear incentive contract assigning weights \( \alpha_i \) to own-firm’s profits and \( \beta_i \) to rival firm’s profits, and with a fixed compensation \( k_i \). Firm i’s profit is

\[ \pi_i (q_i, q_j) = (A - bq_i - aq_j - c_i) q_i + \varepsilon, \]  

and firm i’s managerial compensation is

\[ w (k_i, q_i, q_j) = k_i + \alpha_i \pi_i + \beta_i \pi_j. \]  

The manager chooses quantities to solve the following problem

\[ q_i^{**} = \arg \max_{q_i} E \left[ w (k_i, q_i, q_j^{**}) \right], \]  

where \( q_j^{**} \) is the Cournot equilibrium quantity chosen by firm j’s manager. Lemma 1 gives the equilibrium quantities.
Lemma 1 The industry equilibrium quantities are:

$$q_{i}^{**} = \frac{a_{j} (\beta_{i} + \alpha_{i}) (A - c_{j}) - 2b \alpha_{i} (A - c_{i})}{a^{2} (\beta_{i} + \alpha_{i}) (\beta_{j} + \alpha_{j}) - 4b^{2} \alpha_{i} \alpha_{j}},$$

for $i = 1, 2$ and $j \neq i$.

Shareholders use backwards induction and infer the second stage equilibrium quantities for each triplet $(k_{i}, \alpha_{i}, \beta_{i})$ that they choose. Thus, firm $i$’s shareholders choose $(k_{i}, \alpha_{i}, \beta_{i})$ to solve

$$\max_{k_{i}, \alpha_{i}, \beta_{i}} \left[ E \left( \pi_{i} \left( q_{i}^{**}, q_{j}^{**} \right) \right) - w_{i} \left( k_{i}, q_{i}^{**}, q_{j}^{**} \right) \right],$$

subject to the wage scheme (3) and the participation constraint, $w_{i} \geq \underline{w}$. This problem can be simplified by eliminating $k_{i}$ using the fact that $w_{i} = \underline{w}$ at the optimum. The solution is given in Proposition 1.

Proposition 1 Shareholders of each firm design contracts that include peer performance evaluation. The optimal compensation ratios are $\alpha_{i}/\beta_{i} = z_{i}$, for any $\beta_{i} \neq 0$.

Letting $\rho = b/a$,

$$z_{i} = -2 \frac{(\rho^{2} - 1) (A - c_{i})}{\rho (A - c_{j}) - (A - c_{i})} + 1.$$  

(7)

It is not optimal to set $\beta_{i} = 0$.

Notice that it is not in the shareholders’ interest to set $\beta_{1} = \beta_{2} = 0$. Hence, packages that include relative compensation measures are optimal. Moreover, independently of cost heterogeneity, relative performance schemes do not distinguish the separate roles of the own-price elasticity parameter, $b$, from that of the product substitutability parameter, $a$.

Absent firm heterogeneity, the impact of product market substitutability on relative performance is $dz_{i}/d\rho = dz_{j}/d\rho = -2$. In this case, both firms choose to harden product market competition. Proposition 2 presents the result with cost heterogeneity.
Proposition 2  Without loss of generality, let $c_1 < c_2$. When product market substitutability increases ($\rho$ decreases), firm 2 chooses $\frac{dz_2}{d\rho} < 0$. Depending on the degree of cost heterogeneity, firm 1 may choose $\frac{dz_1}{d\rho} > 0$. 

Proposition 2 shows that firms can react in different ways to increases in product market substitutability (lower $\rho$). Assuming $z_i = \alpha_i/\beta_i < 0$, firm 2 hardens market competition in response to an increase in product market substitutability by putting a relatively more negative weight, $\beta$, in the rival firm’s profit. In contrast, firm 1 may prefer to soften product market competition by choosing a less negative $\beta$. Thus, in this industry, either both firms choose to adjust in identical fashion to changes in product market substitutability or the more efficient firm differentiates itself by giving incentives to soften competition.

Figure 1 illustrates Proposition 2. The Appendix describes the construction of the incentive regions $I_1$ and $I_2$ and demonstrates that region $I_2$ is not empty, at least for values of $\rho$ close to 1. Region $I_1$ is composed of the pairs $(c_2, c_1)$ for which firms behave in a similar fashion. Region $I_2$ gives all other combinations.

Figure 1 shows that for sufficiently large cost differences (region $I_2$), shareholders of low cost firms give incentives to soften competition in response to increases in product substitutability. This is never the case for shareholders of high cost firms. To get some intuition, note that as product substitutability increases the price impact of the rival firm’s quantity increases relative to the price impact of the own firm’s quantity. This negative effect can be ameliorated if shareholders of firm 1 give incentives to reduce competition. This is only optimal if firm 1 is sufficiently more efficient.

---

3 With identical costs, $z_i = -2\rho - 1 < 0$ because $\rho > 0$. With differential costs, it is not possible to analytically sign $z_i$, but $z_i < 0$ in all numerical examples with positive expected profits.
3 Conclusion

Cost heterogeneity is an important source of profit dispersion among firms. This paper shows that incentive compensation contracts can vary significantly across firms with different cost structure.

There are two main predictions. First, shareholders of more efficient firms may want to design incentive contracts that differ from those offered by less efficient firms. To the extent that larger firms are more efficient, I conjecture that the incentive contracts offered to firms within the same industry vary with firm size. Second, in industries that can be characterized by Cournot competition, low cost firms may soften competition whereas high cost firm always harden competition in response to increased product substitutability.
4 Appendix

The appendix provides the proofs of the results in the main text.

Proof of Lemma 1. The manager’s problem is to solve (4). The first order condition can be rewritten to yield the best reply function

$$q_i = \frac{A - a(\beta_i + \alpha_i)q_j - c_i}{2b}.$$ \hspace{1cm} (8)

Tedious algebra reveals that \((q_i^{**}, q_j^{**})\) solves the system of two equations in two unknowns composed by the equation above on \(q_i\) and a symmetric one for \(q_j\). □

Proof of Proposition 1. Problem (6) yields the first order conditions

$$\frac{\partial q_i}{\partial \alpha_i} (A - 2bq_i - aq_j - c_i) = aq_i \frac{\partial q_j}{\partial \alpha_i},$$ \hspace{1cm} (9)

and

$$\frac{\partial q_i}{\partial \beta_i} (A - 2bq_i - aq_j - c_i) = aq_i \frac{\partial q_j}{\partial \beta_i}. \hspace{1cm} (10)$$

It can be shown that these two equations are identical and give only a solution to \(z_i = \alpha_i/\beta_i\). Together with firm j’s first order condition, I solve for \((z_1, z_2)\). Tedious algebra leads to the expressions in the text.

To show that it is never optimal to set \(\beta_i = 0\) (or \(\beta_j = 0\)), note that the manager’s first order condition when \(\beta_i = 0\) is

$$A - 2bq_i - aq_j - c_i = 0.$$ \hspace{1cm} (11)

Inserting this into the shareholder’s first order conditions (9), and (10), gives \(q_i = 0\) (noting that \(\frac{dq_i^{**}}{d\beta_i} \neq 0\) when evaluated at \(\beta_i = 0\)). □

Proof of Proposition 2. Without loss of generality, let \(c_2 > c_1\). Simple computations give

$$\frac{dz_i}{d\rho} = \frac{-2(\rho - 1)^2(A - c_j) + 4\rho(c_j - c_i)}{[\rho(A - c_j) - (A - c_i)]^2}(A - c_i).$$
First note that \( A > c_2 > c_1 \). That \( A > c_2 \) (and \( A > c_1 \)) is needed to ensure positive profits. Thus, \( dz_2/d\rho \) is always negative. As for the more efficient firm, the derivative \( dz_1/d\rho \), is negative iff

\[
-2(\rho - 1)^2(A - c_2) + 4\rho(c_2 - c_1) < 0. 
\]

This condition need not hold if the cost differential is sufficiently large relative to \( \rho \) as I now show. Consider the pairs \((c_i, c_j)\) that satisfy condition (12)

\[
g(c_2) \equiv -\frac{(\rho - 1)^2}{2\rho}(A - c_2) + c_2 < c_1.
\]

Clearly, \( g(c_2) = 0 \) when \( c_2 = \frac{(\rho - 1)^2}{(\rho - 1)^2 + 2\rho} A \). Also, \( g'(c_2) \) is constant and equal to \( 1 + \frac{(\rho - 1)^2}{2\rho} \). Therefore, \( g(.) \) intersects the 45° line. Using \( g(.) \), \( dz_1/d\rho \) is positive iff \( c_1 < g(c_2) \), that is to the right of the locus \( g(c_2) \) or in region \( I_2 \) in Figure 1.

At the point where \( c_1 = c_2 \) the condition implies that \( c_2 = A \). Therefore, at that point and, in fact for any \( (c_1, c_2) = (c_1, A) \), expected profits for firm 2 are negative (see 2). Therefore, by continuity, there is a zero-isoprofit curve that passes to the left of the vertical line \( (c_1, A) \), because profits for firm 2 must also be negative when \( c_2 \) is close enough to \( A \).

Finally, I show that region \( I_2 \) is not an empty set when \( \rho \) is close to 1. To do that, I show that equilibrium quantities, prices and profits are all positive at \( (c_1, c_2) = (0, \frac{(\rho - 1)^2}{(\rho - 1)^2 + 2\rho} A) \). This point defines the starting point of \( g \) and is in the boundary of \( I_2 \). Note that because \( g'(c_2) > 1 \), there exists a sufficiently small \( \varepsilon > 0 \) such that the point \( (c_1, c_2) = (\varepsilon, \frac{(\rho - 1)^2}{(\rho - 1)^2 + 2\rho} A + \varepsilon) \) lies inside \( I_2 \). Therefore, by continuity, \( (c_1, c_2) = (\varepsilon, \frac{(\rho - 1)^2}{(\rho - 1)^2 + 2\rho} A + \varepsilon) \) yields positive equilibrium quantities, prices and profits as well.

Using (7) yields \( z_1 = -2\rho^2 - 1 \) and \( z_2 = -3 \) and from (5) I obtain \( q_1 = \frac{3A_0}{2\rho^2} \frac{2\rho^4 + \rho^2 + 1}{(2 + 6\rho^2)(\rho^2 + 1)} \) and \( q_2 = (2\rho^2 + 1) \frac{36\rho^2 - 6\rho^4 + 15\rho^4 - 9\rho^3 + 4\rho^2 - 3\rho + 1}{2(4 + 12\rho^2)(\rho^2 + 1)} \). Using (1), \( p_1 = A\frac{6\rho^6 - 6\rho^4 + 15\rho^4 - 9\rho^3 + 4\rho^2 - 3\rho + 1}{(4 + 12\rho^2)(\rho^4 + \rho^2)} \) and \( p_2 = A\frac{-6\rho^6 + 12\rho^5 - 4\rho^4 + 7\rho^3 + \rho - 2}{(4 + 12\rho^2)(\rho^4 + \rho^2)} \). It can be verified by evaluating \( p_1 \) that \( p_1 > c_1 = 0 \). Also, \( q_2 > 0 \) if and only if \( \rho > 1/3 \). Finally, for \( p_2 - c_2 > 0 \) a necessary and sufficient
condition is

\[ 2\rho \left( -6\rho^6 + 12\rho^5 - 4\rho^4 + 7\rho^3 + \rho - 2 \right) > (\rho - 1)^2 (3\rho + 9\rho^3 + 6\rho^6 + 4\rho^4 + 2). \]

The condition holds for \( \rho \in (0.7, 1.65) \). Since values for \( \rho < 1.0 \) are not realistic because they imply that the demand for a firm’s product is more sensitive to the rival’s output than its own, a more realistic condition is \( \rho \in (1, 1.65) \). ■

References


Figure 1: Firm 1 is more efficient. In region $I_1$, both firms harden market competition as product substitutability increases. In region $I_2$, the more efficient firm softens market competition instead.