THE TIMING OF EDUCATION

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Abstract
We study an assignment-with-investment model to highlight a tradeoff between investment in human capital before (ex ante system) and after (ex post system) matching on the labor market. The ex post system is better at coordinating investment within firms whereas the ex ante system is better at reducing mismatches. We further show that the ability to transfer surplus within firms affects mismatches and the relative performance of the two systems. At high degrees of transferability, they are equivalent. But when transferability is very low, the ex post system outperforms the ex ante system, although with moderate transferability the reverse is true. (JEL: I21, I28, J31, J42)

1. Introduction
For most people, education is the most significant investment they will make in their lifetimes. But the fruits of an education don’t depend only on the individual making the investment. Rather, they are typically jointly determined by the educations of other individuals (co-workers, firm managers), often unseen and unknown until long after the investment is made: Thus sorting in the labor market is a crucial determinant of the private as well as social returns to an individual’s educational investment. The efficiency of this sorting process has implications for the design of educational systems, which has come under renewed scrutiny in many countries in the face of increased global competition.

The private returns that influence the investment decision will be determined in part by how easily the social returns can be shared within the organization in which one produces. In many situations, those returns can be shared only imperfectly: Incentive problems, liquidity constraints, and “behavioral” considerations...
limit the flexibility of organizations to divide the pie without affecting its size. There are also reasons to believe that this sort of flexibility is decreasing, or at least changing, as the world economy becomes more integrated. Agency problems associated with outsourcing or the dissolution of implicit contracts are examples of reduced flexibility in sharing that can arise from globalization.  

Several literatures have studied from different points of view how imperfections within firms affect returns to investments and therefore the levels of investments that are made (Grossman and Hart 1986; Acemoglu and Pischke 1999). As Legros and Newman (1996, 2002, 2004a) have shown, imperfections within firms are also potential sources of mismatches on the labor market. In particular, a change in the transferability of surplus within firms may modify the way agents sort themselves into firms, and, as we show here, this also affects their incentives to invest.  

The distortion brought by mismatches on the labor market is potentially a function of when agents invest in education, timing that is often influenced by structural conditions and also by educational policy. There appears to be considerable heterogeneity across countries in the timing of education. OECD data show that the age at which tertiary education is acquired varies quite a bit across countries. Data points in Figure 1 are the 20-50-80 quantiles for different OECD countries. For instance, in France, 80% of the individuals acquire tertiary education before they are 20, whereas in Switzerland only 20% are less than 20 (and 80% are less than 29 or 40 depending on the country).

This suggests that in the same countries tertiary education is achieved after having entered the labor market, while in others it is achieved beforehand. The data in Figure 2 support this interpretation: It presents the conditional probabilities that an individual is working given that she is going through tertiary education, by age bracket. For the 15–19 bracket, this probability is less than 7% for France, 30% for the U.S., and 50% for Switzerland.

Educational systems play a role in influencing this timing. For instance in France most students enter grandes ecoles just after high school and spend one or two years preparing for an entrance competition; this also applies to some of the best French business schools. By contrast, in the U.S., business schools tend to

2. This distinguishes the approach in this paper from previous work by Cole, Mailath, and Postlewaite (2001) and Peters and Siow (2002). The first study matching equilibria under the assumption of perfect transferability, the second under the assumption of strict non-transferability. Both consider ex ante investments.
3. Education policies, whether in the form of direct financing of schools, subsidies to special programs, grants at low interest rates to students, mandatory schooling, or minimum standards, affect the incentives of agents to invest in education and also the time at which they acquire education. Firms can also coordinate or even finance the investment in education by their workers, either by executive education, on-the-job training, or direct subsidies for tertiary education.
4. Figure 1 uses table C2.1 and Figure 2 uses table C4.1 from OECD (2003).
value labor market experience among applicants. Our main point in this paper is that evaluating features of educational systems, such as the timing of investments, cannot be done without considering the flexibility of firms to distribute surplus that is indirectly produced by those systems. As firm flexibility and educational policy are both under pressure from changing market forces, these issues are linked not just theoretically but practically as well.

2. Model

The economy is populated by a continuum of agents. Only half of the population (the skilled) can acquire education and their cost of doing so is $c$, the other half of the population is composed of unskilled agents. Upon investing in education, agents become educated ($E$), otherwise they are not educated ($N$). All characteristics are observable. The parametric assumptions on agents’ education acquisition costs are

$$c \in (1, 1.5).$$

5. Of course, the timing of education is not just due to the design of the educational system: Other reasons may have to do with financial constraints or varying opportunity costs over the business cycle.

6. Admitting asymmetric distributions of low-cost and high-cost agents changes our analysis only marginally. Types’ payoffs are determined uniquely by relative scarcity as the shorter market side gets all the surplus. The reader is referred to our paper Gall, Legros, and Newman (2005) for a discussion.
A firm consists of two agents jointly producing output. Total surplus in a firm is a function of whether agents are educated or not, \( y(E, E) = 8 \); \( y(E, N) = 7 \); \( y(N, N) = 4 \).

Note that \( y \) satisfies decreasing differences. It follows from (1) that total welfare is maximum when firms consist of a skilled agent who invests in education and an unskilled agent who does not invest in education. Total welfare is then \( W_{FB} = (7 - c)/2 \).

We assume now a simple form of imperfection within firms: If output is \( y \), then the share of an agent must lie in the interval \( [(y/2) - b, (y/2) + b] \). In other words, starting from equal sharing an agent is able to transfer at most \( b \) to the other agent.\(^7\) This non-transferability will prevent efficient coordination on educational choices and on firm formation: As we will see sometimes educational choices are consistent with the first-best—all skilled invest—but firm composition is not first-best; at other times, firm composition is the right one—skilled and unskilled agents together—but the educational choices are not first-best.

Agents can invest in education either before or after the labor market opens. Date 0 is the ex ante stage, stage 1 is the labor market clearing, stage 2 is the ex post stage. There is no possibility for an agent at stage 0 to sign a contract with a

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\(^7\) A model of moral hazard in teams with limited liability can lead to such a constraint on shares. Wage rigidity, risk aversion, imperfect insurance, or “behavioral” considerations will also lead to non-transferabilities in firms.
firm. On the labor market agents match in firms of size 2. Competition takes the form of wage contracts, contingent on characteristics of agents and possibly on future investment in education.

We will consider two situations:

*Ex ante*—When all education must be acquired before the labor market opens (e.g., mandatory education). Because agents have already invested and because the costs are sunk and do not affect future production, contracts will be wage contracts. Matching will take the form of pairs \((E, E)\), \((E, N)\), or \((N, N)\).

*Ex post*—When all education must be acquired after the labor market opens, when agents are already in firms (e.g., on-the-job training, continuing education). Matching in the labor market at stage 1 is based on whether agents are skilled or not. Contracts define a wage structure that can be made contingent on output as well as on whether the agent has acquired education.

This ignores the possibility that agents choose when they want to acquire education. The general analysis is made in Gall, Legros, and Newman (2005).

We therefore highlight two differences between the ex ante and ex post regimes. First, competition on the labor market is on the basis of educational achievement in the first case and cost of acquiring education in the second case. Second, educational choices are coordinated by the market in the ex ante system while they are coordinated within a firm in the ex post system. In the ex ante regime, education serves as a “ticket” to get the surplus available in firms. By contrast in the ex post regime, agents will coordinate efficiently on educational choices given the constraint on surpluses that the labor market imposes. As we will show, it follows that the role of non-transferabilities within firms has a different effect on educational choices and sorting in the two systems.

2.1. Ex Ante

In the ex ante regime, an educated will be induced to form a firm with a non-educated agent only if the wage offered is greater than 4, the equal treatment payoff an educated agent can obtain by belonging to a \((E, E)\) firm. Because the maximum wage an educated can obtain in a \((E, N)\) firm is \(3.5 + b\), it is necessary that \(b \geq 1/2\). If \(b < 1/2\), the equilibrium is the same as in the case \(b = 0\) and agents segregate: Skilled invest and form \((E, E)\) firms while unskilled are in \((N, N)\) firms.

If \(b > 1/2\), competition in the labor market precludes having \((E, E)\) firms, because unskilled non-educated agents can transfer \(t \in (\frac{1}{2}, b)\) in order to attract \(E\) agents into \((E, N)\) firms. In equilibrium, sorting must be stable and educational choices must be efficient. For stability, non-educated agents must not prefer
being in an \((N, N)\) firm, that is, their wage in an \((E, N)\) firm must be large enough: \(3.5 - t \geq 2\), or \(t \leq 1.5\). Therefore, transfers from \(N\) to \(E\) must be \(t \in [0.5, \min\{b, 1.5\}]\). For educational choices, consider the skilled agents. If \(t = 1/2\), the wage of non-educated agents in an \((E, N)\) firm is 3, which is also their total surplus (because they do not invest), the surplus of an educated agent is their wage of 4 minus the cost of education \(c\) and because \(4 - c < 3\), a skilled agent would prefer not to acquire education. In order to align incentives, the wage of 3 of a non-educated in an \((E, N)\) firm must be obtained with a probability less than one. This will happen when there is excess supply of non-educated, that is when some skilled agents do not invest in education.

Let \(\alpha\) be the measure of skilled agents who invest; \(\alpha \leq 0.5\) and \(\alpha/(1 - \alpha)\) is the probability that a non-educated agent forms a firm with an educated agent. The expected wage of a non educated agent is \(w(\alpha) = (\alpha(3.5 - t) + (1 - 2\alpha)2)/(1 - \alpha)\). Therefore skilled agents prefer weakly to acquire education if and only if \(3.5 + t - c \geq w(\alpha)\), that is,

\[
\alpha \leq \alpha(t) = \frac{1.5 + t - c}{3 - c}.
\]

The bound in (2) is lower than the total measure of skilled agents (0.5) only if the transfer is \(t \leq c/2\). Hence when \(b \leq c/2\), only a measure \(\alpha(b) < 0.5\) of skilled agents invest, and there are \(\alpha(b)\) firms \((E, N)\) and a measure \(0.5 - \alpha(b)\) of \((N, N)\) firms forming at the labor market stage.

**Proposition 1.** In the case of ex ante education, a market equilibrium is described by a measure \(\alpha(b)\) of skilled agents acquiring education, a transfer \(t\) from \(N\) to \(E\), and the set of firms. The set of market equilibria is the following:

- For **low transferability** \((b < 1/2)\), \(\alpha(b) = 1/2\), there are equal measures of \((E, E)\) and \((N, N)\) firms, \(t = 0\): there is efficient investment in aggregate, but mismatching implies overinvestment within \((E, E)\) firms and aggregate underproduction.
- For **moderate transferability** \((b \in (0.5, c/2))\), \(\alpha(b) = (1.5 + b - c)/(3 - c) < 1/2\), there are \(\alpha(b)\) firms \((E, N)\) and \((1/2 - \alpha(b))\) of \((N, N)\) firms; within \((E, N)\) firms educated agents receive an additional transfer \(t = b\): there is aggregate underinvestment and underproduction.
- For **high transferability** \((b \geq c/2)\), \(\alpha(b) = 1/2\), all firms are \((E, N)\) and \(t \in [c/2, (b, 1.5)]\): Equilibrium is first-best efficient.

Aggregate welfare is increasing in the degree of transferability \(b\).8

8. For \(b = 0.5\), there are two possible equilibria, corresponding to the low- and moderate-transferability equilibria described in Proposition 1.
2.2. Ex Post

By the equal treatment property of the labor market equilibrium, agents with the same cost of acquiring education must be treated symmetrically. In a firm consisting of skilled agents, the maximum equal treatment surplus is attained when each agent invests with equal probability while the other does not invest. This can be implemented via a correlation device with values 0 and 1: When the value is 0, the first agent is asked to invest and the second does not invest, and when the value is 1, the roles are reversed. Because $3.5 - c > 2$, the agent who is asked to invest will do so, and because $4 - c < 3.5$, the agent who is asked not to invest will also be obedient. Hence, the best equal treatment surplus for skilled agents is $v_s = (7 - c) / 2$ whereas for unskilled, the best equal treatment payoff is $v_u = 2$. These surpluses are lower bounds on surpluses for skilled and unskilled agents for any value of $b$.

Consider now a firm consisting of a skilled and unskilled agent. A contract specifies the wage $w(y)$ to the skilled agent and the probability $\beta$ with which this agent is expected to invest in education. For a given $b$, we have $w(y) \in [y/2 - b, (y/2) + b]$. It is immediate that incentive compatible contracts $(\beta, w)$ satisfy

$$\beta = 1 \text{ as } w(7) - w(4) > c$$

and

$$\beta \in [0, 1] \text{ as } w(7) - w(4) = c.$$  \hspace{1cm} (3)

The expected surplus of a skilled and an unskilled agents are, respectively, $v_s = \beta(w(7) - c) + (1 - \beta)w(4)$ and $v_u = \beta(7 - w(7)) + (1 - \beta)(4 - w(4))$. Such a firm will arise only if $v_s \geq v_u$. This leads to the constraints

$$\beta(w(7) - c) + (1 - \beta)w(4) \geq (7 - c)/2$$  \hspace{1cm} (4)

and

$$\beta(7 - w(7)) + (1 - \beta)(4 - w(4)) \geq 2.$$  \hspace{1cm} (5)

From (4), we can have $\beta = 1$ only if $w(7) - c \geq (7 - c)/2$, or, because $w(7) \leq 3.5 + b$, when $b \geq c/2$. As $b = c/2$, skilled agents get the equilibrium surplus $v_s$, whereas unskilled agents get a surplus of $v_u$. As $b$ increases, the Pareto optimal contracts specify $\beta = 1$ and $t \in [c/2, c/2 + 1]$. When $b < c/2$, we cannot have $\beta = 1$. By (3) we have $\beta \in (0, 1)$ only if $c = w(7) - w(4)$, but then, the surplus of a skilled agent is equal to $w(4)$; now from (4) and $w(4) \leq 2 + b$, we need $b \geq (3 - c)/2$ which is impossible when $b < c/2$ because it would imply that $c/2 > (3 - c)/2$, or $c > 3/2$, which contradicts (1). Hence when $b < c/2$, agents segregate.
PROPOSITION 2. In the case of ex post education,

- For low to moderate transferability \( b < c/2 \), agents segregate. In firms consisting of skilled agents, the agents correlate on a device where each bears the cost of investment with equal probability. Investment is efficient within firms, but there is aggregate underinvestment and underproduction due to mismatch.

- For high transferability \( b \geq c/2 \), there is a measure \( 1/2 \) of firms consisting of a skilled agent and an unskilled agent; the skilled agent invests with probability one and receives a transfer from the unskilled agent of \( t \in [c/2, \min\{b, 1.5\}] \). The first-best is achieved.

2.3. Comparison

Using the two propositions, it follows that the ex post system will lead to a higher total welfare only if \( b \) is smaller than \( 1/2 \), whereas the ex ante system dominates when \( b \in [1/2, c/2] \), and the two systems are equivalent when \( b \geq c/2 \). This comparative static result is a consequence of the role of education in each system. Although the ex post system has an advantage at coordinating educational investments within firms, the ex ante system is better at aligning educational incentives with marginal returns on the labor market; this marginal return of investment is however a function of the sorting on the labor market, which can be inefficient.

Hence, the ex ante system performs best when there is “enough” transferability within firms.

When there is a low degree of transferability, both systems suffer from mismatch. To move from segregation to the more efficient regime of mixed firms would require a departure from equal sharing, which is very costly under low transferability. The ex post system at least coordinates on education and so saves resources that are wastefully spent in the ex ante regime.

But this greater coordination efficiency is the ex post regime’s undoing when transferability increases to moderate levels. For now the ex ante system moves away from segregation, whereas the large payoff to the skilled that obtains under the ex post system becomes a hindrance to compensating them for the extra burden of education that they must assume in mixed firms. Thus mismatch remains a problem for the ex post regime. Ex ante firms gain more from increased monetary transferability than ex post firms because the latter already a form of imperfect transferability through their allocation of the investment burden.

3. Concluding Remark

Many countries now view themselves as undergoing crises in education. The foregoing analysis raises the possibility that reductions in the flexibility of firms to share surplus (brought on for instance by globalization) may be part of the reason.
Educational systems that resemble our ex ante case may have worked well in the past but may no longer be optimal if transferability has decreased.

References


