Competing for Ownership[*]

Patrick Legros[†] and Andrew F. Newman[‡]

Abstract

We develop a tractable model of the allocation of ownership and control within firms operating in competitive markets. The model shows how scarcity in the market translates into ownership structure inside the organization. It identifies a price-like mechanism whereby local liquidity or productivity shocks propagate, leading to widespread organizational restructuring. Among the model's predictions: firms will become more integrated when the terms of trade become more favorable to the short side of the market, when the liquidity of the poorest firm increases sufficiently relative to the mean, and following a uniform increase in productivity. Shocks to the first two moments of the liquidity distribution

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[‡]Boston University and CEPR.

have multiplier effects on the corresponding moments of the distribution of ownership structures.

1 Introduction

In the neoclassical theory of the firm, market signals affect choices of products, factor mixes, and production techniques. If labor becomes scarce, wages rise, and firms substitute machines for workers. Firm behavior, in turn, influences the market, and through it, other firms: if a labor-saving technique is found by some firms, wages fall and other firms switch to less capital-intensive techniques. The neoclassical firm remains the backbone of much of economic analysis because it is so readily incorporated into the study of feedback effects like these.

The modern theory of the firm emphasizes contractual frictions and organizational design elements such as monitoring technologies, task allocations, asset ownership, and the assignment of authority and control. In so doing, it has led to breakthroughs in our comprehension of institutions as different as the modern corporation and the sharecropped farm. But despite the theory's formative purpose – to understand the nature of firms in market economies – as well as evidence that firms restructure themselves in response to market conditions or the behavior of other firms, [1] there are few models that can take account of the effects of the neoclassical feedbacks on organizational design.

[1] To mention just two examples: the wholesale restructuring of relations between U.S. auto makers and their suppliers in the 1980s was likely triggered by entry of Japanese firms into the U.S. market; on a smaller scale, decision rights over the outfitting of truck cabs or the accompaniment of drivers by their spouses during hauls have recently shifted from trucking firms to their drivers in response to the growth of wages in the construction industry.

The purpose of this paper is to provide a simple framework for this kind of analysis. We focus on the structure of ownership and control, understood here (as in Grossman and Hart 1986), as an allocation of residual decision rights among a firm's stakeholders.[2] The model illuminates how scarcity in the market translates into control inside the firm and how changes in the fundamentals of some firms can spill over to economy-wide reorganizations.

The basic setup is a two-sided matching model, where the sides represent two types of production units, each consisting of a manager and a collection of assets. Firms comprising one unit of each type form through a competitive matching process that determines, for each matched pair, a contract specifying its ownership structure.

Once a firm has formed, a series of noncontractible management decisions – one for each asset – must be taken, after which output is realized and the relationship ends. The organization must be designed to strike a compromise between productivity (managers share the firm's profit) and the private costs of managing; because of the noncontractibility, this can only be accomplished by a (re-)allocation of the rights to own or control the various assets.[3]

In general, the more assets a manager owns, the better-off he is, since he

[2]This distinguishes the present paper from earlier work such as Calvo and Wellisz (1978) and Legros and Newman (1996), which focused on the general equilibrium determination of monitoring and incentives. See also Garicano and Rossi-Hansberg (2006).

[3]The literature following Grossman and Hart (1986) and Hart and Moore (1990) tends to distinguish *ownership* from *control* by identifying ownership with a party's right to exclude others' access to an asset, whereas control is applied to most other decisions concerning its use. In the static environment without renegotiation that we study here, there is little meaningful distinction between these concepts and so we use the terms interchangeably.

can ensure that more decisions go in his preferred direction. But because these decisions impose both profit and private cost externalities on the other manager, different organizational designs generate different *levels* of total surplus for the firm as well as different *divisions* of that surplus between its managers.

A crucial attribute of the environment we analyze is that liquidity – instruments such as cash that can be transferred costlessly and without any incentive distortions – is scarce. Managers have quasi-linear utility, so liquidity transfers are the preferred means of reallocating surplus between them. But when liquidity is in short supply, a large transfer of surplus must occur via an organizational distortion, that is, a reassignment of control. This feature generates a key role for competitive analysis. The equilibrium outcome can no longer be identified with the surplus-maximizing allocation of ownership; instead, the market-determined division of the surplus is needed to pin down the organizational outcome. In our model, for instance, a high degree of integration – in which one manager controls the preponderance of assets – arises only if there is a sufficiently uneven division of surplus in his firm.

The model highlights two distinct effects that arise from a change in fundamentals such as liquidity endowments or technology. The first is an *internal effect*, various forms of which have been studied in the literature on ownership: the surplus that each partner obtains from a given contract is a function of the characteristics of the partners in a relationship, in particular the amount of liquidity they have and the production technology available to them. In our model, more liquidity in the firm enlarges the set of feasible payoffs for the two managers by increasing transferability, though it does not enlarge their set of production possibilities because there is no need to acquire productive assets from outside the partnership. Higher productivity not only enlarges the payoff sets by expanding production possibilities, but also increases transferability, by inducing managers to increase the weight of profit (which can be shared) relative to private cost (which cannot) in their decisions. Hence, a positive shock to a firm's liquidity or productivity will enable it to accomplish surplus division more efficiently and reduce organizational distortions.

But the impact of that shock can extend well beyond the firm that first experiences it. After a positive shock, a manager has a greater "ability to pay" for a partner than he did before. He may thus bid up the terms of trade in the matching market, and firms that have not benefited from the shock may have to restructure in order to meet the new price. Thus, there is also an *external effect*: "local" shocks can propagate via the market mechanism, leading to widespread reorganization.

The market equilibrium of our model turns out to be amenable to a Marshallian supply-demand style of analysis that makes the role of the external effect especially transparent. Suppose, for instance, that one side of the market represents automobile manufacturers selling in the U.S. market and the other side represents their suppliers. An increase in the number of manufacturers due to entry from abroad will reduce the share of surplus accruing to the auto makers. This will entail a transfer of control to the suppliers, and many manufacturer–supplier relationships will become less integrated: a smaller fraction of the assets will be controlled by the auto maker's manager.

Furthermore, whereas the internal effects of positive shocks to liquidity and technology are similar (they both decrease integration), the external effects differ. A uniform increase in the liquidity level of all agents lowers the degree of integration in all firms (the internal effect dominates the external effect). By contrast, a uniform shock to productivity increases the degree of integration in all firms (the external effect dominates the internal effect). Moreover, small shocks may have large consequences. For instance, a unit change in mean liquidity produces a larger than unit change in the mean degree of integration: there is an "organizational multiplier" effect. As we show in Section 3, the model can also handle the analysis of more complex changes in the liquidity endowments or in productivity.

Our model of the determination of ownership structure is inspired by Grossman and Hart (1986). However, we depart from their analysis in three respects. First, as in Hart and Moore (1990), we allow for a richer set (in fact, a continuum) of ownership structures rather than the two (integration and nonintegration) discussed by Grossman and Hart. This feature yields tractability for competitive analysis, as well as the flexibility to capture the broad array of control allocations displayed by real firms (for examples, see Lerner and Merges 1998 on biotechnology R&D alliances; Arruñada, Garicano, and Vázquez 2001 on automobile dealerships; and Blair and Lafontaine 2005 on fast-food franchises). Second, as have several recent papers (e.g. Hart and Holmström 2002; Aghion, Dewatripont, and Rey 2004; Baker, Gibbons, and Murphy 2006), we abstract from the holdup problem by dropping ex ante investments and assuming instead that ex post decisions are not contractible. Our purpose in doing so is to make the surplus transfer role of ownership especially transparent: the set of feasible decisions is unaffected by who owns an asset; therefore, awarding ownership of more assets to one manager unambiguously raises his payoff.

The third and most important departure is the assumption that liquidity is scarce. The corporate finance literature beginning with Aghion and Bolton (1992) has already highlighted what we have termed the internal effect of limited liquidity on the allocation of control: given the division of surplus, raising a contractual party's liquidity endowment will tend to give him more control and to increase the relationship's efficiency. What is new here is the identification and analysis of the external effect: limited liquidity implies that a firm may modify its control rights allocation, at a possible efficiency cost, in response to changes in the liquidity (or technology) of *another* firm. This effect would also be present for many other specific models of ownership and organizational design; all that is important is that the payoff frontier *not* reflect transferable utility, which scarce liquidity helps to guarantee.

2 Model

We consider an economy in which there are two types of production units, indexed by $1, 2$. Each unit consists of a risk-neutral manager and a collection of assets that he will have to work with in order to produce. We have in mind competitive outcomes, so we suppose there is a large number of production units: each side of the market is a continuum with Lebesgue measure. The type-1 units are represented by $i \in I = [0, 1]$ and the type-2 units are represented by $j \in J = [0, n]$ with $n < 1$; thus, the type-2's are relatively scarce. Production units may either operate on a stand-alone basis, in which case they earn an outside option (normalized to zero), or cooperate in pairs comprising one unit of each type, in which case they can generate strictly positive surplus.

Many interpretations are possible. The two types of manager might be supplier and manufacturer with the assets being plant and equipment; a chain restaurateur and franchising corporation, where some of the assets are reputational; or a firm and its workforce, for which the assets might be thought of as tasks.

In an individual production unit, an asset's contribution to profit depends on a planning decision made by one of the managers, not necessarily the one who will have to operate it. Planning decisions are not contractible, but the right to make them can be allocated via contract to either manager. For simplicity we assume that planning choices (e.g., choosing the background music for a retail store) are costless. Though potentially beneficial for profits (some music is likely to induce consumers to make impulse purchases), such choices affect the private cost of later operations (the music may be unpleasant for the store's floor manager).

The ith type-1 manager will have at her disposal a quantity $l_1(i) \geq 0$ of cash (or "liquidity"), which may be consumed at the end of the period and may be useful in contracting with managers of the opposite type; for the type-2 managers, the liquidity endowment is $l_2(j)$. The indices i and j have been chosen in order of increasing liquidity. When discussing a generic production unit or its manager, we shall usually omit the indices.

2.1 The Basic Organizational Design Problem

2.1.1 Technology and Preferences

A manager seeks to maximize her expected income (including the initial liquidity) less the private costs of operating the enterprise; we refer to this payoff net of initial liquidity as the manager's *surplus*.

The collection of assets in the type-1 production unit is represented by a continuum indexed by $k \in [0, 1)$; the type-2 assets are indexed by $k \in [1, 2]$. An asset's contribution to profit is proportional to the planning level $q(k)$, where $q(k) \in [0, 1]$.

Planning decisions contribute to the firm's performance as follows. The firm either succeeds, generating profit $R > 0$, with probability $p(q)$; or it fails, generating 0, with probability $1 - p(q)$. Here $q : [0, 2] \to [0, 1]$ are the planning decisions. The success probability functional is

$$p(q) = \gamma \int_0^2 q(k)dk,$$

where $\gamma < 1/2$ is a technological parameter. Define productivity $A = \gamma R$.

Either manager is capable of making planning decisions. There is no cost to making a plan, but there is a (private) operating cost to the manager who subsequently works with an asset: the 1-manager bears cost $c(q(k)) = q(k)^2/2$ for $k \in [0, 1)$ and zero cost for $k \in (1, 2]$; similarly the 2-manager bears cost $c(q(k))$ on $[1, 2]$ and zero on $[0, 1)$. The aggregate costs to each manager are

$$C_1(q) = \int_0^1 c(q(k))\, dk,\ C_2(q) = \int_1^2 c(q(k))\, dk.$$

This is the source of the cost externality that may arise from reallocating control: the manager's disutility of operating the asset is increasing in $q(k)$

regardless of whether she has chosen it. In a manufacturing enterprise, for example, q could index choices of possible parts or material inputs ordered by the value they contribute to the final product, and $c(q)$ could represent the cost of managerial attention devoted to overseeing assembly, supervising workers, and so on, where higher-value inputs require greater management effort.[4]

2.1.2 Contracts

We have already made the following contractibility assumptions.

- The *right to decide* $q(k)$ is both alienable and contractible.
- The decisions q are never contractible.
- The costs $C_i(q)$ are private and noncontractible.

A contract (ω, t) specifies the allocation of ownership ω and liquidity transfers t made from 1 to 2 before any planning or production takes place. Since the liquidity levels of the two types are respectively l_1 and l_2, we must have

[4] Note that we are assuming symmetry in the technology and cost between the two managers; any difference that emerges between the two sides will be due only to a difference in scarcity. One could extend the model to allow for asymmetries in cost, productivity, or initial number of assets. For instance, if $C_2 \equiv 0$, then a firm is basically a principal–agent relationship. If the type-2 is interpreted as "capital," the model could be viewed as a static version of a financial contracting problem, as in Aghion and Bolton (1992). Assuming that one type is more productive that the other allows one to to ask the kind of questions addressed by Grossman and Hart (1986) and Hart and Moore (1990) concerning who should (as against who does) own the assets. For some applications – e.g., firms and workers – it might be appropriate to assume that one type (firms) initially owns and bears the cost of most of the assets.

$t \in [-l_2, l_1]$. The ownership allocation ω is the fraction of assets reassigned to one of the managers. The type-1 manager owns assets in $[0, 1-\omega)$ and the type-2 in $[1-\omega, 2]$, where $-1 < \omega \leq 1$.

Because our focus here is on allocations of control rights, we simplify matters by ignoring the effects related to variations in the sharing of profits. Instead, we assume that each manager gets half of the realized output – that is, he gets $R/2$ if output is R, and 0 if output is 0. This is a simple representation of the constraints faced by real firms in the use of incentive pay. Similar assumptions have been used elsewhere in the literature (e.g., Hart 1983; Holmström and Tirole 1998), and in Appendix A, we show that it can be derived as a consequence of a moral hazard problem.[5]

This leaves out a logical possibility:[6] the managers might use a third party "budget breaker" who will pay the firm if there is success and will be paid out of the firm's available liquidity if there is failure. Using third parties in this way may improve efficiency, but only if the third party gets more when the firm fails than when it succeeds. Apart from the undesirable incentive problems this

[5]One can also relax the assumption and allow for a rich set of budget-balancing sharing rules to yield predictions on the interplay between ownership allocations and profit shares. The modified model of the firm can easily be embedded in our framework, leading to only minor modification of the results in Section 3. See Legros and Newman (2007).

[6]There are three others. First, that the managers "swap" assets: in additon to ω, which indicates how many of 1's assets are shifted to 2, the contract would have an additional variable ψ indicating how many of 2's assets are shifted to 1. Second, that the managers pledge their liquidity to increase the total revenue available *after* the output is realized. Third that agents use external finance (i.e., sign debt contracts). We show in Appendix A that none of these possibilities can improve on contracts as we define them.

creates (the third party may want the firm to fail), such a modification would not change the basic message of this paper.[7]

When $\omega = 0$, each manager retains ownership of his original assets, and, following the literature, we refer to this situation as *nonintegration.* As ω increases beyond 0, we have an increasing degree of integration (the fraction of the assets owned by 2's is growing) until with $\omega = 1$ we have full integration. (The symmetric cases with $\omega < 0$ correspond to 1-ownership; with scarce 2's and zero outside options for the 1's, ω will turn out to be positive in equilibrium, and we focus on this case in what follows unless noted otherwise.) Since ω not only describes the ownership structure but also provides a scalar measure of the fraction owned by one party, we shall often refer to its (absolute) value as the *degree of integration* of the firm.

2.1.3 The Feasible Set for a Firm

Given the incentive problems arising from contractual incompleteness, it should come as no surprise that the first-best solution (in which $q(k) = A$ for all k) cannot be attained. For tasks $k \in [0, 1)$, a type-1 manager who makes the planning decision will underprovide q because she bears the full cost of the decision but receives only half of the revenue benefit. In contrast, a type-2 manager who makes the planning decision will overprovide q because increasing q increases expected output at no cost to himself.

[7] It would, however, make the analysis more complex; in particular, the simple supply-demand analysis we perform here would be replaced by a matching problem. For an example of the use of third parties in the formation of firms when there are liquidity constraints, see Legros and Newman (1996).

The profit shares are fixed and so, in the absence of liquidity, the only way to allocate surplus is to modify the degree of integration ω. Given a contract (ω, t), the two managers subsequently choose q noncooperatively to maximize their corresponding objectives:

$$u_1(\omega, t) = \max_{q(k)\in[0,1], k\in[0,1-\omega]} \frac{A}{2}\int_0^2 q(k)dk - \frac{1}{2}\int_0^1 q(k)^2\, dk - t,$$
$$u_2(\omega, t) = \max_{q(k)\in[0,1], k\in(1-\omega,2]} \frac{A}{2}\int_0^2 q(k)dk - \frac{1}{2}\int_1^2 q(k)^2\, dk + t.$$

It is straightforward to see that manager 1 will set $q(k) = A/2$ on the assets $k \in [0, 1-\omega)$ that she controls and that manager 2 will choose $q(k) = 1$ for $k \in [1-\omega, 1)$ and $q(k) = A/2$ for $k \in [1, 2)$. Then, the payoffs associated to a contract (ω, t) are

$$u_1(\omega, t) = 3A^2/8 - \omega(2-A)^2/8 - t, \tag{1}$$

$$u_2(\omega, t) = 3A^2/8 + \omega A(2-A)/4 + t. \tag{2}$$

Because reallocating control rights does not affect the feasible set of planning decisions, a manager gaining control of additional assets cannot be worse-off.[8]

Proposition 1 *A manager's payoff is nondecreasing in the fraction of assets he controls.*

Note that the Pareto frontier when there is no liquidity (so that $t = 0$) is

$$v_2 = \begin{cases} -\alpha v_1 + (\alpha+1)\frac{3}{8}A^2, & \text{if } v_1 \le \frac{3}{8}A^2, \\ -\frac{1}{\alpha}v_1 + (\frac{1}{\alpha}+1)\frac{3}{8}A^2, & \text{if } v_1 \ge \frac{3}{8}A^2, \end{cases} \tag{3}$$

[8]This invariance of the feasible set to transfers of control stems from the absence of investments made before q is chosen; in particular, it extends to cases in which there are noncontractible investments ex post and/or in which sharing rules are flexible. See Legros and Newman (2007).

where $\alpha = 2A/(2-A) < 1$ measures the degree of payoff transferability. Observe that the total surplus generated by a contract ω, $u_1(\omega,t)+u_2(\omega,t)$, is maximal at $\omega = 0$ (nonintegration) provided

$$A < 2/3. \tag{4}$$

We shall focus on this case.[9]

If managers have no liquidity then $t = 0$ and, as 1's payoff decreases, the number of assets that 2 owns (weakly) increases. At the same time, total surplus is decreasing; thus it is fair to say that here *reallocations of ownership are used to transfer surplus, not merely to generate it.* Observe that this mode of surplus transfer is less efficient than transferring cash, so any liquidity that the managers can spare will be used first to meet the surplus division demanded by the market before they transfer ownership.

When agents of types 1 and 2 have liquidity l_1 and l_2, the set of feasible payoffs they can attain via contracting is defined by equations (1) and (2) along with uncontingent transfers that do not exceed the initial liquidities. Given the risk neutrality of the managers, ex ante transfers do not affect total surplus; in particular, we have $u_1(\omega,t) = u_1(\omega,0)-t$ and $u_2(\omega,t) = u_2(\omega,0)+t$. Figure 1 illustrates a typical feasible set, which we denote $U(l_1,l_2)$, when agents have liquidity l_1 and l_2. The dark segments represent the frontier in the absence of liquidity transfers. The surplus maximum occurs at the kink, where $\omega = 0$; we have indicated a point **a** on this frontier corresponding to a transfer ω^* of control to manager 2. Point **b** indicates the surplus levels of 1 and 2 after 1

[9] When $A > 2/3$, the frontier is nonconcave and, absent lotteries, the most efficient organization will entail giving nearly full control to one of the managers.

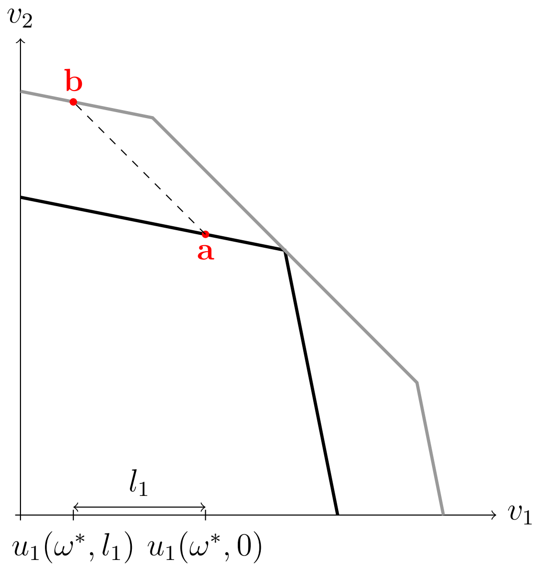

Figure 1: Feasible Set

also transfers all of her liquidity l_1; the gray segments trace the entire frontier available to this pair of managers.

2.2 Market Equilibrium

Market equilibrium is a partition of the set of agents into coalitions that share surplus on the Pareto frontier; the partition is stable in the sense that no new firm could form and strictly improve the payoffs to its members. The only coalitions that matter are singletons and pairs (which we call "firms") consisting of a single type-1 production unit $i \in I$ and a single type-2 production unit $j \in J$. The excess supply of type-1 production units means that there is at least a measure $1-n$ of type-1 managers who do not find a match and so obtain a surplus of zero. Stability requires that no unmatched type-1 manager can bid up the surplus of a type-2 manager and still receive a positive surplus. Necessary conditions for this are that all type-2 managers be matched and the surplus of each be no less than $u_2(0,0) = 3A^2/8$. As is apparent from the construction of the feasible set, if $v_2 > u_2(0,0)$ then payoffs on the Pareto frontier are achieved by transferring the type-1's liquidity only; that is, type-2's liquidity does not matter. Thus all 2's are equally good, as far as a type-1 manager is concerned, and must therefore receive the same surplus.[10]

This "equal treatment" property for the type-2 managers is an important simplification relative to most matching models in which there is heterogeneity on both sides of the market. Identify the set of firms F with the index of the type-1 manager in the firm; "firm i" indicates that the firm consists of the ith type-1 production unit and a type-2 manager.

Definition 1 *An equilibrium consists of a set of firms $F \subset I$ with Lebesgue measure n, a surplus v_2^* received by the type-2 managers, and a surplus function $v_1^*(i)$ for the type-1 managers such that the following conditions hold.*
(i) (Feasibility) For all $i \in F$, $(v_1^*(i), v_2^*) \in U(l_1(i), 0)$. For all $i \notin F$, $v_1^*(i) = 0$.
(ii) (Stability) For all $i \in I$, all $j \in J$, and all $(v_1, v_2) \in U(l_1(i), l_2(j))$, either $v_1 \leq v_1^*(i)$ or $v_2 \leq v_2^*$.

[10] If in firm (i,j) type-2 j has a strictly larger surplus than type-2 j' in firm (i',j'), then the firm (i,j') could form and both i and j' could be better-off because the Pareto frontier is strictly decreasing. Note that if the 1-types have large enough outside options (or are more scarce than the 2-types), their liquidity does not matter although the liquidity of type-2 managers does. It can also be shown that only 2-types' liquidities matter when $A > 2/3$.

2.2.1 Characterizing Market Equilibrium

Since the type-2 managers have the same equilibrium payoff, we can reason in a straightforward demand-and-supply style by analyzing a market in which the traded commodity is the type 2's. We construct the demand as follows. The amount of surplus a 1 is willing *and able* to transfer to a 2 depends on how much liquidity she has. A 1's willingness to pay is the value of the problem

$$\max_{(\omega,t)} u_2(\omega,t)$$
$$\text{s.t. } u_1(\omega,0) \geq t,\ t \in [0,l_1].$$

In the contract (ω,t), the type 1 manager gets $u_1(\omega,t)+l_1$; the opportunity cost of the contract is to be unmatched and receive l_1. Hence the manager is willing to contract when $u_1(\omega,t) \geq 0$ which is equivalent to the condition stated since $u_1(\omega,t) = u_1(\omega,0) - t$. Simple computations show that the solution to this problem is

$$(\omega,t) = \begin{cases} (0, \frac{3}{8}A^2) & \text{if } l_1 \geq \frac{3}{8}A^2, \\ (\frac{3A^2-8l_1}{(2-A)^2}, l_1) & \text{if } l_1 < \frac{3}{8}A^2. \end{cases} \tag{5}$$

The willingness of a type-1 manager to pay for matching with a type-2 manager is then

$$W(i) = \begin{cases} \frac{3}{4}A^2 & \text{if } l_1(i) \geq \frac{3}{8}A^2, \\ \frac{3}{8}A^2 + (\frac{3}{4}A^2 - 2l_1(i))\frac{A}{2-A} + l_1(i) & \text{if } l_1(i) \leq \frac{3}{8}A^2. \end{cases} \tag{6}$$

Since the frontier has slope magnitude less than unity above the 45° line, and since $l_1(i)$ is increasing in i, the willingness to pay of i is nondecreasing in i. If type-2 managers must receive a payoff of v_2, then the measure of type-1 managers who are willing and able to pay this price is

$$D(v_2) = 1 - \min \{i \in [0,1] : W(i) \geq v_2\}.$$

The supply is vertical at n, the measure of 2's (see Figure 2). Equilibrium is at the intersection of the two curves: this indicates that n of the 1's are matched, as claimed previously, and that the marginal 1 is receiving zero surplus.

Proposition 2 *The equilibrium set of firms is* $F = [1-n, 1]$, *and the equilibrium surplus of type-2 managers is*

$$v_2^* = \min \left\{ \frac{3}{4} A^2, W(\bar{l}_1) \right\},$$

where $\bar{l}_1 = l_1(1-n)$.

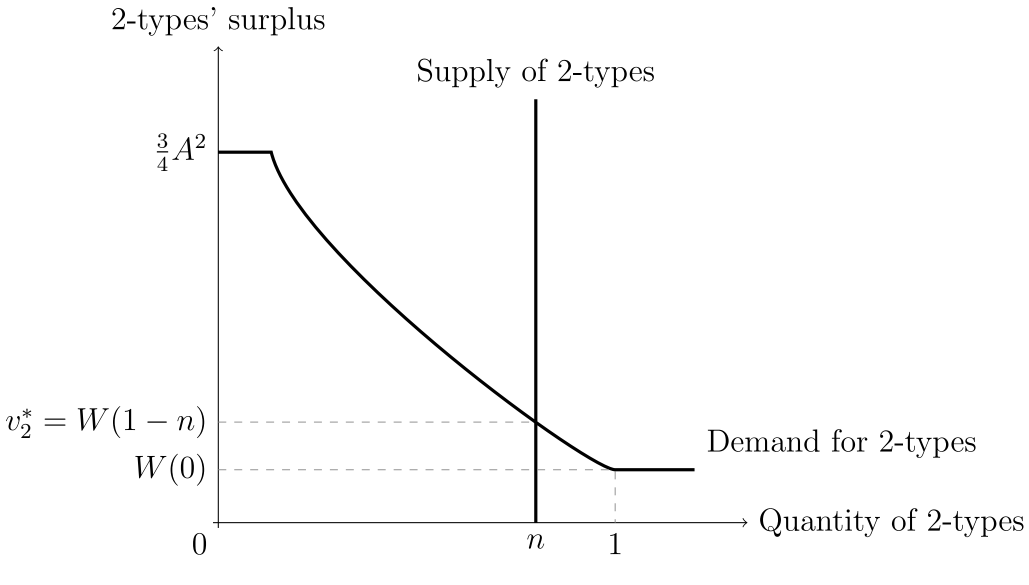

Figure 2: The Market for Ownership

We are mainly interested in situations in which $\bar{l}_1 < 3A^2/8$.[11] In this case, the equilibrium surplus of type-2 managers is $v_2^* = W(\bar{l}_1) < 3A^2/4$. The marginal type-1 manager $1-n$ has a surplus of 0, but the inframarginal type-1 managers with liquidity $l_1 > \bar{l}_1$ will be able to generate a positive surplus for themselves because they can transfer more liquidity than the marginal 1. The surplus of an inframarginal 1 when the price is v_2^* is the value of the problem

$$\max_{\omega} u_1(\omega, t) + l_1$$
$$\text{s.t. } u_2(\omega, 0) + t = v_2^*,\ t \leq l_1.$$

The solution to this problem is $\omega(v_2^*, l_1)$ and $t(v_2^*, l_1)$, where

$$\begin{aligned} \omega(v_2^*, l_1) = 0, \text{ and } t(v_2^*, l_1) = v_2^* - \frac{3}{8}A^2 \text{ if } l_1 \geq v_2^* - \frac{3}{8}A^2 \\ \omega(v_2^*, l_1) = 4\frac{v_2^* - \frac{3}{8}A^2 - l_1}{A(2-A)}, \text{ and } t(v_2^*, l_1) = l_1 \text{ if } l_1 \leq v_2^* - \frac{3}{8}A^2. \end{aligned} \tag{7}$$

In this model there is a piecewise linear relationship between liquidity, degree of integration, level of output, and managerial welfare. We account for the internal and external effects by noting that the degree of integration is a nonincreasing function of liquidity and a nondecreasing function of the price v_2^*. If a firm's liquidity increases, it will tend to become less integrated – unless this effect is overcome by a concomitant increase in the price v_2^*, which in turn depends on the liquidity and the technology available in the economy. Thus, a systematic study of the effects of shocks must take account of the endogeneity of v_2^*, which we do in the next section.

[11] If $\bar{l}_1 \geq 3A^2/8$ then the second-best efficient outcome with all firms nonintegrated is obtained, since each matched type-1 manager is able to pay $3A^2/8$ to the type-2 manager. Observe that in this case the equilibrium surplus of all type-1 managers is zero.

Lemma 1 *The degree of integration $\omega(v_2^*, l_1)$ is piecewise linear: it is increasing in v_2^* and decreasing in l_1 when $l_1 < v_2^* - 3A^2/8$, and it is equal to zero when $l_1 \geq v_2^* - 3A^2/8$.*

3 Comparative Statics of Market Equilibrium

In equilibrium there will typically be variation in organizational structure across firms, and this is accounted for by variation in their characteristics. In particular, "richer" firms are less integrated and generate greater surplus for their managers.[12]

But more liquidity overall can also lead to *more* integration: if the marginal firm's liquidity increases then v_2^* rises, and possibly by more than an inframarginal firm's gain in liquidity. As a result, the inframarginal firm may become more integrated, and indeed it is possible that the economy's average level of integration may increase via this external effect.

We shall consider three types of shocks that may lead to reorganizations in the economy: changes in the relative scarcity of the two types, changes in the distribution of liquidity, and changes in the productivity parameter A.

3.1 Relative Scarcity

In order to isolate the external effect our first comparative statics exercise involves changes in the tightness of the supplier market – that is, in the relative

[12]Holmström and Milgrom (1994) emphasize a similar cross-sectional variation in organizational variables. In their model, the variation reflects differences in technology but not differences in efficiency relative to their potential, since all firms are surplus maximizing. In contrast, here the variation stems from differences in liquidity and reflects differences in organizational efficiency.

scarcities of 1's and 2's.

Suppose that the measure of 2's increases, as from the entry of foreign downstream producers into the domestic market. Then, just as in the standard textbook analysis, we represent this by a rightward shift of the supply schedule: the price of 2's decreases. Indeed, as n increases, the marginal type-1 liquidity decreases because $l_1(1-n)$ is decreasing with n. What is different from the standard textbook analysis, of course, is that this change in price entails (widespread) corporate restructuring.

Let $F(n)$ be the set of firms when there is a measure n of type 2 firms. As n increases to $\hat{n}$, there is a new equilibrium set $F(\hat{n})$,with $F(n) \subset F(\hat{n})$: new firms are created after the increase in supply, but we can suppose that previously matched managers stay together. The surplus of *all* type-1 managers in $F(n)$ increases. Firms in $F(n)$ will restructure (decrease ω) in response to a reduction in the equilibrium value of v_2^*. The analysis is similar in the opposite direction: a decrease in the measure of 2's leads to an increase in v_2^*. Thus we have the following result.

Proposition 3 *In response to a small increase (decrease) in the measure of type-2 managers, the firms originally (remaining) in the market become less (more) integrated.*

It is worth remarking that if the relative scarcity changes so drastically that the 2's become more numerous, then type-1 managers get the preponderance of the surplus and tend to become the owners; the analysis is similar to what we have already seen, with the types 1 and 2 reversed. The point is that the owners of the integrated firm gain control because they are scarce, not because it is efficient for them to do so: in this sense, organizational power stems from market power.

For increases in demand by the 1's, effects similar to those generated by a reduction in the supply of 2's might be expected: v_2^* would rise, leading to increased integration. However, this analysis is incomplete. An increase in demand for 2's most likely stems from entry of new firms (which in turn entails a change in the liquidity distribution among the active firms) and from increases in productivity (e.g., "skill-biased technical change"). Therefore, a general analysis of the effects of changes in relative scarcity requires separate consideration of the effects of changes in liquidity and productivity. We provide this in the next two subsections.

3.2 Liquidity Shocks

Evaluating changes in the liquidity distribution is complicated by the interplay of internal and external effects. The dependence of ownership structure ω on the type-1 liquidity l_1 and on the equilibrium surplus v_2^* was summarized in Proposition 2 and Lemma 1. Equipped with these results, we can derive some characterizations and simple comparative statics of the distribution of ownership structures.

First, if one is interested in minimizing the degree of integration in the economy (this maximizes the surplus), then it is clear from (7), Proposition 2, and Lemma 1 that the marginal liquidity should be minimized – this minimizes the equilibrium price. In addition, the liquidity of the inframarginal firms should be maximal. Since the function $\omega(v_2^*, l_1)$ is globally convex in l_1, it follows that the mean degree of ownership is minimal when all firms have the same level of liquidity. More generally, there is a simple description of the set of distributions that minimize average integration in the economy.

Proposition 4 *Let L be the average liquidity among the type-1 managers. The degree of integration is minimized when the marginal 1 has zero liquidity and when the distribution of liquidity among the inframarginal 1's has support in* $[0, 3\alpha A^2/8]$ *for* $L < 3\alpha A^2/8$ *and support in* $[3\alpha A^2/8, \infty)$ *for* $L > 3\alpha A^2/8$.

We now consider how the distribution of ownership depends on the distribution of liquidity. To simplify, we restrict attention to liquidity distributions in which all 1's are liquidity constrained and belong to firms with a positive ω. In Appendix B, we consider the general case in which a positive measure of 1's are in nonintegrated firms.

Let $G(l)$ be the distribution of liquidity among the type-1 managers and $\bar{l}_1$ the marginal liquidity. Let $\mu = (1/n)\int_{\{l \geq \bar{l}_1\}} l dG(l)$ and $\sigma^2 = (1/n)\int_{\{l \geq \bar{l}_1\}} (l - \mu)^2 dG(l)$ be, respectively, the mean and variance of liquidity of the inframarginal 1's. When all firms are at least partially integrated, the linearity of the degree integration in l implies a monotonic relationship between the first two moments of the distribution of liquidity and those of the distribution of ownership.

Proposition 5 *The mean and the variance of the degree of ownership are*

$$E(\omega) = \omega_0 + a\bar{l}_1 - b\mu \quad \textit{and}$$

$$Var(\omega) = b^2\sigma^2,$$

respectively, where $\omega_0 = 3A^2/(2-A)^2$, $a = 4(2-3A)/(A(2-A)^2)$, *and* $b = 4/(A(2-A))$.

The dependence of the mean degree of ownership on the liquidity of the marginal type reflects the external effect, since a higher liquidity at the marginal relationship implies a higher degree of integration in other firms. When all firms choose a positive ω, the variance of ω depends only on the variance of liquidity. We show in Appendix B that if a positive measure of 1's are not liquidity constrained, then the variance of ownership also depends on the marginal and mean liquidities as well as on the variance.

Ownership structure is sensitive to liquidity. Since $A < 2/3$ we know that b exceeds 4.5. Hence, a unit increase in the mean liquidity that does not raise $\bar{l}_1$ leads to more than a 4-fold decrease in the average level of integration; a unit increase in the variance of liquidity generates an over 20-fold increase in the variance of integration. We are not aware that such a "multiplier effect" of fundamentals on organizational structure has been previously noted in the literature.

It is easy to compare outcomes for two distributions of liquidity G and H. Suppose that the marginal level of liquidity is larger at H than at G: $G(\bar{l}_1^G) = H(\bar{l}_1^H) = 1-n$ implies $\bar{l}_1^G < \bar{l}_1^H$. It follows that a 2's price is greater with H than with G; in fact, from (6), $v_2^{*H} = v_2^{*G} + (1-\alpha)(\bar{l}_1^H - \bar{l}_1^G)$. Hence each type-1 who is inframarginal uses a greater degree of integration with H than with G. However this is not incompatible with a decrease in the average degree of integration if the average liquidity increases enough: the internal effect must compensate for the external effect. This is formally stated in our next proposition.

Proposition 6 *Consider two distributions of liquidity G and H for which all firms choose $\omega > 0$.*

(i) The mean degree of ownership is lower with H than with G if and only if

$$\underbrace{(1-\alpha)(\bar{l}_1^H - \bar{l}_1^G)}_{\textit{change in price}} < \underbrace{\mu^H - \mu^G}_{\textit{change in average liquidity}}.$$

(ii) The variance of the degree of ownership is lower in H than in G if and only if the variance of liquidity is lower with H than with G.

A special case worth highlighting is that of positive and nondecreasing shocks to each 1's liquidity. Note that a uniform shock in which every type-1 manager receives the same increase to her endowment is a special case, as is a multiplicative shock in which the percentage increase to the endowment is the same for all 1's. The shock will increase the type-1 manager's willingness to pay, which (via the internal effect) reduces the degree of integration. Yet it will also increase the equilibrium surplus of the type-2 managers, which (via the external effect) has the opposite impact.

However, it is a simple matter to demonstrate that in this case the internal effect dominates: more liquidity implies less integration. For instance, if every type-1 has ε more liquidity, the price increases by $(1-\alpha)\varepsilon$, while the average liquidity increases by ε; hence the condition of the proposition holds because $\alpha < 1$. With nondecreasing shocks, if the marginal liquidity goes up by ε then the average liquidity increases by more than ε, so the condition of the proposition is once again satisfied. Of course, shocks that are negative and nonincreasing yield the opposite changes in surplus and organization.

Corollary 1 *Under positive and nondecreasing shocks to the liquidity distribution of 1's, the aggregate degree of integration decreases.*

To maintain this conclusion, the proviso that shocks be monotonic can be relaxed, but not arbitrarily. Positive shocks alone are not enough, and having more liquidity in the economy may actually imply that there is higher overall degree of integration. Intuitively, if the positive shock hits only a small neighborhood of the marginal 1's, then the price v_2^* will increase and the inframarginal unshocked firms will choose to integrate more in response to the increase in v_2^*.

Proposition 7 *There exist first-order stochastic dominant shifts in the distribution of type-1 liquidity that lead to more integration.*

For the proof, see Appendix B.

3.3 Technology and Demand Shocks

The external effect outlined in the previous section offers a propagation mechanism whereby shocks that initially affect only a few firms may nevertheless entail widespread reorganization. Empirically, this implies that explaining a particular reorganization need not require finding a smoking gun in the form of a liquidity change *within* that organization; instead, the impetus for such reorganization may originate elsewhere in the economy. The same logic applies to other types of shocks, particularly those to productivity. These are often thought to be the basis of large-scale reorganizations such as merger waves (Jovanovic and Rousseau 2002).

We model a (positive) productivity or technological innovation as an increase in A. This could come from an increase in the success probability parameter γ or in the output generated when there is success; it could also be interpreted as a demand shock that raises the profit R via an increase in the product price (particularly if all firms experience an increase in A).

It will facilitate the Marshallian analysis if we think of the technology as inhering in the type-1's. Suppose that all firms in the initial economy have the same technology; after a shock, a subset of them (an interval $[i_0, i_1]$) have access to a better technology $\hat{A} > A$. We restrict ourselves here to considering "small" shocks in the sense that $\hat{A} < 2/3$.

Raising A modifies the game that managers play: it is clear from equations (1) and (2) that both managers obtain a larger surplus from a given contract (ω, t). Hence the feasible set expands and the 1's willingness to pay also increases. What is perhaps less immediate is that there is also more transferability within the firm.

Lemma 2 *Let A be the initial productivity. After a positive productivity shock:*
(i) The feasible set expands.
(ii) For any $t < 3A^2/8$, the degree of integration solving $u_1(\omega, t) = 0$ increases.
(iii) There is more transferability in the sense that the slope of the frontier is steeper in the region $v_2 \geq v_1$ when A increases.

Proof. (i) Given (4), we can differentiate (1) and (2) with respect to A and thus show that, for any contract (ω, t), both $u_1(\omega, t)$ and $u_2(\omega, t)$ are increasing in A.
(ii) Use (5).
(iii) The absolute value of the slope of the frontier in the region $v_2 \geq v_1$ is $\alpha = 2A/(2 - A)$, which is also increasing in A. ■

The willingness to pay (6) depends on the technology available to the firm.

Because firms differ in their technology, we write:

$$W(i) = \min\left\{\frac{3}{4}A_i^2, \frac{3}{8}A_i^2 + (\frac{3}{4}A_i^2 - 2l_1(i))\frac{A_i}{2-A_i} + l_1(i)\right\} \tag{8}$$

$$\text{with } A_i = \begin{cases} A & \text{if } i \notin [i_0, i_1]\,, \\ \hat{A} & \text{if } i \in [i_0, i_1]\,. \end{cases}$$

Lemma 2(iii) implies that, for a *fixed* equilibrium surplus v_2^*, a shocked firm integrates less because it is able to transfer surplus via ω in a more efficient way. Hence when the "price" of 2's is fixed, positive technological shocks lead to less integration in the economy.

However, Lemma 2(ii) implies that the price will increase when the marginal firm is shocked. Since by (iii) there is more transferability with ω, liquidity has less value: the inefficiency linked to the use of integration is lower and so integration is a better substitute for liquidity transfers. This implies that type-1 managers find it more expensive, in terms of liquidity, to "buy" control. Thus, technological change that increases the 2s' equilibrium surplus is a force for integration. Unshocked firms certainly integrate more; for shocked firms, we will show that they benefit internally from the technological shock but the countervailing effect of an increase in the 2s' equilibrium surplus dominates. The net effect is toward more integration *for all firms in the economy* if the marginal firm is a shocked firm. Other results are contained in the following proposition.

Proposition 8 *Suppose positive technology shocks occur to the type-1 agents indexed by* (i_0, i_1).

(i) (Inframarginal shocks) If $i_0 > 1 - n$*, then the shocked firms become less*

integrated and the unshocked firms remain unaffected.

(ii) (Marginal shocks) If $1-n \in (i_0, i_1)$ *and if* $1-n$ *is still the marginal type-1 agent, then the equilibrium price increases and all firms – shocked and unshocked – integrate more.*

(iii) (Uniform shocks) If there is a uniform shock to the technology ($i_0 = 0$, $i_1 = 1$) *then each firm integrates more.*

Thus, the effect of small positive productivity shocks depends on what part of the economy they affect. If they occur in "rich" firms (case (i)), then only the innovating firms are affected, and they become less integrated. But innovations that occur in "poor" firms (case (ii)) may affect the whole economy and in the opposite direction: even firms that don't possess the new technology become more integrated.

It is worth noting that Corollary 1 and Proposition 8(iii) imply that uniform liquidity and technology shocks have opposite effects: uniform increases in liquidity reduce integration whereas uniform improvements in technology increase it. In this sense, the external effect of productivity shocks is more powerful than that for liquidity shocks.

If the 1's are differentiated by technology alone, then entry of type-2 production units will lead to a marginal relationship with a smaller value of A than before entry. By Lemma 2, v_2^* decreases and all incumbent firms will choose a lower level of ω.

This argument can be generalized if type-1 managers are differentiated by their liquidity endowments as well as by their productivity. Order the 1's by their willingness to pay rather than their liquidity – a higher willingness to pay indicates a higher liquidity or a higher value of A but not necessarily both. For a given A and l_1, the frontier is decreasing in type-1's payoff and in ω; therefore entry by the 2's unambiguously decreases ω for all incumbent firms, and 1's gain control.

Proposition 9 *Suppose that 1's are differentiated both by liquidity and technology. Then, entry of type-2 production units will lead to more control by originally matched 1's.*

Unlike in Proposition 3, this need not imply that all original firms become less integrated. When the 1's differ in productivity, it is possible that some (high-productivity, inframarginal) firms will have $\omega < 0$ initially (1 controls some of 2's assets) and that entry of 2's reduces ω further (i.e., increases integration of those firms).

4 Illustrations

4.1 Entry in Supplier and Product Markets: Automobiles

Until the 1980s, large U.S. automobile manufacturers maintained arm's-length relationships with their suppliers, usually setting specifications for parts without their involvement and then awarding production contracts via competitive bidding. In contrast, Japanese automotive firms had long embraced a "partnership" model with their suppliers.

Following a wave of foreign direct investment by Japanese firms in the United States, Chrysler started reorganizing its relationship with suppliers and eventually involved them as almost equal partners in product and process development. Other U.S. manufacturers soon followed suit. This change in supplier relations has been linked to the threat posed by the entry of Japanese firms; by their dominance in the market for small cars, which was the fastest-growing segment following successive oil crises; and the comparatively greater quality of Japanese cars, which seemed attributable to the close cooperation with suppliers for design and development (see, for instance, Dyer 1996).

In terms of our model, interpret type 2 as the car manufacturers, type 1 as the suppliers, and $\omega > 0$ as the degree of control that car manufacturers have in their relationships with suppliers. A move from the old arm's-length relationship to the partnership arrangement is characterized by a decrease in ω as the suppliers gain control over aspects of the design and production process. The entry of Japanese producers into the United States both affected the product market, corresponding to a fall in revenue parameter R (and therefore A) for all firms, and the supplier market, corresponding to an outward shift of the supply of 2's. (The Japanese firms relied in part on U.S. suppliers; there was not concomitant entry into the supplier side, as suggested in part by the reductions in the number of suppliers each U.S. automaker dealt with following its reorganization.)

Given these interpretations, the change in supplier relations in the U.S. auto industry is consistent with our model. From Proposition 8(iii), the model predicts that reduced profitability for U.S. auto makers (a uniform decrease in R) leads to a decrease in ω for all U.S. firms. The increased competition in the supplier market from the Japanese (rightward shift in the supply of 2's) will have the same effect (Proposition 3).

If one looks only at the relationship between one auto firm (Chrysler, say) and its supplier, then assuming a fall in R due to Japanese competition would provide little guidance in predicting how ω would change. Indeed, by expression (7), a decline in A actually implies an increase rather than a decrease in ω unless v_2^* falls enough. Only the full "general equilibrium" analysis provided in Proposition 8 (iii) tells us that v_2^* does fall enough to bring about the observed decline in ω.

4.2 Technological Shocks outside the Industry: Trucking

In the 1980s and 1990s, the U.S. trucking industry experienced a shift away from drivers who owned their own trucks toward employee drivers. This organizational change has been attributed to technological developments such as on-board computers (OBCs), which offered better monitoring of drivers and greater dispatching flexibility, thereby permitting more efficient use of trucks (Baker and Hubbard 2004).

By the early 2000s, the prevalence of owner operators and use of OBCs had stabilized. More recently, however, the industry has begun to shift some control back to drivers. Between 2004 and 2006, carriers began offering drivers such "perks" as the right to travel with spouses or to outfit their cabs with satellite televisions. Since drivers decide whether and when to exercise these rights, this constituted an increase in their control. The question is why there has been a shift of control allocations in trucking without an apparent technological shift.

A possible answer comes from the observation that an important alternative employment for truckers is construction, which experienced a boom in the early 2000s. Thinking of the drivers now as the 2's and of the construction-cum-trucking firms as the 1's, the construction boom would raise A for the construction firms (considered to be the marginal ones). By Proposition 8(ii), our model predicts a rise in ω (i.e., an increase in the degree of control enjoyed by the drivers). The evidence suggests that participants in the industry understand this perfectly well: firms perceive a "shortage" of drivers (Nagarajan, Bander, and White 2000 – this justifies thinking of drivers as 2's) and both kinds of participants attribute the need to offer perks to the boom in construction (Urbina 2006). The payoffs to drivers in trucking firms (i.e., v_2^*) increases, leading to a rise in ω as a result of the external effect generated by the increase in A in the construction sector.

5 Discussion

If one asks *who gets organizational power in a market economy?*, a tempting answer is "to the scarce goes the power." There is a tradition in the business sociology literature (reviewed in Rajan and Zingales 2001) ascribing power or authority to control of a resource that is scarce within the organization. Similar claims can be found in the economic literature (Hart and Moore 1990; Stole and Zweibel 1996). Our results suggest that organizational power may emanate from scarcity *outside* the organization – that is from market power. Agents on the short side of the market, with the greatest wealth, or with the highest skills will tend to garner more control than other agents. How much power they accrue will depend in part on the market price of partners and thus on the distribution of resources among all agents in the economy, not just those in the organization.

And the lesson must be interpreted with some care: redistribution of a scarce resource may cause the recipient to *lose* power via the external effect (consider an increase in productivity by the marginal manager as in Proposition 8(ii)).

As we discussed, one empirical implication of the external effect is that it may account for organizational change that does not originate inside the organization. While it is clear that legal or regulatory change might influence a firm's ownership structure, the external influences on a firm's organization are much broader than that, and include liquidity, technological, or demand shocks in other firms or industries. We are not aware of attempts to quantify the real-world significance of external effects, but we hope that models such as this one will encourage empirical investigations in that direction.

We now discuss some other implications of the model.

5.1 Interest Rate

We have assumed that the interest rate (the rate of return on liquidity) is exogenous and is not affected by changes in the liquidity distribution or in the technology available to firms. One can easily extend the model to allow for liquidity that yields a positive return through the period of production. Because liquidity in this model is used only as a means of surplus transfer and not as a means to purchase new assets, the effects of this extension can be somewhat surprising. Raising the interest rate means that liquidity transferred at the beginning of the period has a higher value to the recipient than before. Formally, the effect is equivalent to a multiplicative positive shock on the distribution of liquidity, and (by Proposition 1) firms will integrate less if the interest rate increases and will integrate more if the interest rate decreases. If liquidity transfers made in the economy affect the interest rate, then increases in the aggregate level of liquidity may, by lowering interest rates, constitute a force for integration above and beyond that suggested by the example in Proposition 7. These observations suggest that the relationship between aggregate liquidity and aggregate performance is unlikely to be straightforward. Whether the potentially harmful organizational consequences would counter or even outweigh the traditional real investment responses is a question for future research.

5.2 Product Market

If we imagine that all the firms sell to a competitive product market, then the selling price inheres in A, which we have thus far viewed as exogenous. But if instead price is determined endogenously in the product market, then shocks to some firms will be transmitted to the others via the product market as well as the supplier market. In other words, more than just the very poorest firms in the economy may be "marginal." For instance, suppose that a number of perfectly nonintegrated firms innovate. With fixed prices, these firms produce more output but nothing further happens. With endogenous prices, the increased output in the first instance lowers product price. All other firms in the economy treat this exactly like a (uniform) negative productivity shock: they all become less integrated. Thus product market price adjustment has a kind of "amplification" effect on organizational restructuring.

Moreover, organizational decisions may affect the quantity of goods produced and hence the product price. For instance, if R is the price of a single unit of output, then industry output is increasing in the degree of integration. As discussed in Legros and Newman (2006), that the product market (even a competitive one) can be affected by the internal organization decisions of firms has implications for consumer welfare, the regulation of corporate governance, and competition policy.

A Appendix: Contracting

We have defined contracts by (ω, t) and equal sharing of the output ex post. This definition might be restrictive because it ignores the following four potential extensions.

- *Contingent shares.* A contract could specify state contingent revenues $x_i(R)$ and $x_i(0)$ to $i = 1, 2$.
- *Debt contract.* Type 1 borrows B from a financial institution in exchange for a repayment of D after output is realized.
- *Ex post transfers of liquidity.* The total liquidity available in the firm is $L = l_1 + l_2$. This liquidity can be transferred either ex ante or added to the revenue of the firm ex post.
- *Asset swapping.* This is a means of effectively committing the managers to high levels of q. The commitment is worthwhile only if productivity is sufficiently high relative to costs, which will not be the case given our parametric restriction. If assets are to be swapped, then we can characterize the situation via two ownership parameters ψ and ω: manager 1 owns $k \in [0, 1-\omega)$ and $k \in [2-\psi, 2)$, and manager 2 owns the other assets.

We show (i) that the restriction to equal revenue shares imposed in the text can be rationalized by introducing a moral hazard element to the model described there and (ii) that the other extensions then do not expand the set of feasible allocations.

Equal revenue sharing. Suppose that in addition to ω and t, contracts include contingent shares $x_i(R)$ and $x_i(0)$ for manager i if the verified revenue realizations are R and 0 respectively, and (possibly) an uncontingent payment T (possibly in addition to t). Suppose further that a manager has the opportunity to divert revenue (but not T, which might be held in escrow) by choosing an effort $e \in [0, 1]$. If the revenue produced by the firm is R, then with probability e the verified revenue will be R; with probability $1-e$ the verified revenue will be 0, in which case the manager succeeds in diverting cR to herself and $(1-c)R$ is lost $(0 \leq c \leq 1)$. If the revenue is 0, then the verified output will be 0 independently of e. Only one manager has the opportunity to divert, her identity being chosen by nature after q is chosen but before output is realized.

Then, with $e = 1$, i's expected share is $p(q)x_i(R) + (1 - p(q))x_i(0) + T_i$. With $e = 0$, the expected share is $p(q)(cR + x_i(0)) + (1 - p(q))x_i(0) + T_i$. Hence $e = 1$ is optimal for i when $x_i(R) - x_i(0) \geq cR$. Clearly, if $c > 1/2$ then these incentive compatibility constraints cannot hold simultaneously for both managers without violating budget balance. But setting $c = 1/2$ yields

$$x_i(R) - x_i(0) = R/2; \tag{A.1}$$

coupled with our other assumptions (limited liability and risk neutrality), one might aws well assume $x_i(R) = R/2$ and $x_i(0) = 0$.

For smaller values of c, there is scope for unequal shares for the two managers. This case is detailed in Legros and Newman (2007), where it is shown that there is no loss in assuming that $T = 0$ and that debt contracts are weakly dominated by nondebt contracts. Under mild parametric restrictions, there is no gain from asset swapping either.

Suppose then that (A.1) holds. A contract is denoted by

$$((\omega, \psi), (B, D), (x_1, x_2), (t_1, t_2)),$$

where we assume without loss of generality that only 1's incur a debt contract. Let x_i^* be the state-contingent share, equal to $R/2$ in state R and to 0 in state 0. We want to show that there exists a contract $((\hat{\omega}, 0), (0, 0), (x_1^*, x_2^*), (\hat{t}_1, L - \hat{t}_1))$ that leads to payoffs that are weakly greater for both managers. We establish this result sequentially: first by showing that $((\omega, \psi), (B, D), (x_1, x_2), (t_1, t_2))$ is weakly dominated by the contract $((\omega, \psi), (0, 0), (x_1^*, x_2^*), (t_1 + x_1(0), t_2 + x_2(0)))$, where neither debt nor ex post transfers of liquidity are used; and second by showing that this contract is dominated by one in which only part of the assets of type 1 are reassigned to type 2 $((\hat{\omega}, 0), (0, 0), (x_1^*, x_2^*), (\tilde{t}, L - \tilde{t}))$.

No debt and ex post transfers. In a contract $((\omega, \psi), (B, D), (x_1, x_2), (t_1, t_2))$, feasibility requires that $t_1 + t_2 \leq L + B$ and that $t_i \geq 0$ for $i = 1, 2$. We write $t = t_1 + t_2$ and let $T = L + B - t$ be the liquidity that is pledged to the firm. Ex post total revenues are then T and $T + R$. Managers receive state contingent shares $x_i(0), x_i(R)$ that satisfy budget balancing and limited liability: $x_1(0) + x_2(0) = T$, $x_1(R) + x_2(R) = T + R$, $x_i(0) \geq 0$, and $x_i(R) \geq 0$.

If there is a debt contract, then manager 1 must repay $\min\{D, x_1(0)\}$ in state 0 and $\min\{D, x_1(R)\}$ in state R. Since by (A.1) we need $x_2(R) - x_2(0) = R/2$, it follows that $x_1(R) - x_1(0) = R/2$; however, since manager 1 must repay the debt, her effective marginal compensation is

$$x_1(R) - x_1(0) - [\min\{D, x_1(R)\} - \min\{D, x_1(0)\}].$$

This is consistent with (A.1) only if $\min\{D, x_1(R)\} = \min\{D, x_1(0)\}$ or if $D \leq x_1(0)$. In this case, debt is not risky; the creditor makes a nonnegative profit only if $D \geq B$, but then we would need $x_1(0) \geq B$ and so $x_2(0) \leq L+B-t-B = L-t$. It follows that the initial contract $((\omega, \psi), (B, D), (x_1, x_2), (t_1, t_2))$ is weakly dominated by the contract $((\omega, \psi), (0, 0), (x_1^*, x_2^*)(t_1+x_1(0), t_2+x_2(0)))$. Because $\sum_{i=1,2}(t_i + x_i(0)) = L$, there is no liquidity transferred ex-post.

No asset swapping. Finally, consider a contract $((\omega, \psi), (0, 0), (x_1^*, x_2^*), (t, L-t))$ consisting of a swap of assets and ex ante transfers; we denote such contracts by $((\omega, \psi), t)$. Then we have the following Nash equilibrium payoffs:

$$u_1(\omega, \psi, t) = \frac{A}{2}((2-\omega-\psi)\frac{A}{2} + \omega + \psi) - \frac{1}{2}(\omega + (1-\omega)\frac{A^2}{4}) - t;$$
$$u_2(\omega, \psi, t) = \frac{A}{2}((2-\omega-\psi)\frac{A}{2} + \omega + \psi) - \frac{1}{2}(\psi + (1-\psi)\frac{A^2}{4}) + t.$$

Suppose without loss of generality that $t > 0$ and that $u_2(\omega, \psi, t) - t > u_1(\omega, \psi, t) + t$; then we must have $\omega > \psi$.

Let $\omega^0 = \omega - \psi A/(1 - A/2)$; since $A/(1 - A/2) < 1$ and $\omega > \psi$, we have $\omega^0 > 0$. Then $u_1(\omega^0, 0, t) = u_1(\omega, \psi, t)$, whereas $u_2(\omega^0, 0, t) - u_2(\omega, \psi, t) = \psi(2 - A - A^2)/4 > 0$ since $A < 1$. By continuity there exists an $\hat{\omega} < \omega^0$ such that the contract $((\hat{\omega}, 0), t)$ strictly Pareto dominates the contract $((\omega, \psi), t)$. If $u_2(\omega, \psi, t) - t < u_1(\omega, \psi, t) + t$, then a similar argument applies when we decrease the value of ψ appropriately.

B Appendix: Proofs

B.1 Proof of Proposition 4

Recall that the liquidity of the marginal type 1 is denoted $\bar{l}_1$. If $\bar{l}_1 = 0$, note that $v_2^* = W(0) = 3(1+\alpha)A^2/8$. From (7) $\omega(v_2^*, l)$ has a kink at $l = 3\alpha A^2/8$ it follows that for lower values the degree of integration is linear and for larger values it is zero; hence $\omega(v_2^*, l)$ is indeed globally convex in l (we suppress the subscript on l when there is no ambiguity).

Suppose that $L < 3\alpha A^2/8$. Let $\underline{L} = \int_{l<3\alpha A^2/8} l\, dG(l)$ and $\bar{L} = \int_{l>3\alpha A^2/8} l dG(l)$. Observe that, by (7), $\int_{l<3\alpha A^2/8} \omega(v_2^*, l)\, dG(l) = \omega(v_2^*, \underline{L})$ and $\int_{l>3\alpha A^2/8} \omega(v_2^*, l) dG(l) = \omega(v_2^*, \bar{L})$. Hence, $E(\omega) = G(3\alpha A^2/8)\omega(v_2^*, \underline{L}) + (1 - G(3\alpha A^2/8))\omega(v_2^*, \bar{L})$. However, since $\omega(v_2^*, \bar{L}) = 0$ and since ω is globally convex, it follows that $L = G(3\alpha A^2/8)\underline{L} + (1 - G(3\alpha A^2/8))\bar{L}$ implies $E(\omega) > \omega(v_2^*, L)$. This shows that $\bar{L} = 0$ and that the support of G is contained in $[0, 3\alpha A^2/8]$. The same argument applies when $L > 3\alpha A^2/8$.

B.2 Proof of Proposition 5

We know from (7) and Proposition 2 that for a given distribution G, the degree of integration is positive when l belongs to $[\bar{l}_1, v_2^* - 3A^2/8)$. In this case we can write $\omega(v_2^*, l) = \omega_0 + a\bar{l}_1 - bl$, where $\omega_0 = 3A^2/(2-A)^2$, $a = 4(2-3A)/(A(2-A)^2)$, and $b = 4/(A(2-A))$; note that $a/b = 1-\alpha$. Let $\kappa = G(v_2^* - 3A^2/8) - G(l)$ be the measure of firms choosing a positive ω.

(i) Let $\mu = \frac{1}{\kappa}\int_{\bar{l}_1}^{v_2^* - 3A^2/8} l\, dG(l)$ be the conditional mean among firms choosing a positive ω. We have

$$\begin{aligned}
E(\omega) &= \int \omega(v_2^*, l)\frac{dG(l)}{n} \\
&= \frac{1}{n}\int_{\bar{l}_1}^{v_2^*-3A^2/8} (\omega_0 + a\bar{l}_1 - bl)\, dG(l) \\
&= \frac{\kappa}{n}(\omega_0 + a\bar{l}_1) - b\int_{\bar{l}_1}^{v_2^*-3A^2/8} l\,\frac{dG(l)}{n} \\
&= \frac{\kappa}{n}(\omega_0 + a\bar{l}_1 - b\mu)
\end{aligned}$$

when all firms choose $\omega > 0$ and $\kappa = n$, leading to the expression in the lemma.

(ii) Let $\sigma^2 = \int_{\bar{l}_1}^{v_2^*-3A^2/8}(l-\mu)^2 dG(l)/\kappa$ be the variance of liquidity among the liquidity-constrained 1's – that is, those that will be in firms with $\omega > 0$. Direct computations show that the variance of ownership is

$$\begin{aligned}
Var(\omega) &= \int \left[\omega(v_2^*, l) - E\left[\omega\right]\right]^2\ \frac{dG(l)}{n} \\
&= \int_{\bar{l}_1}^{v_2^*-3A^2/8} \left[\omega(v_2^*, l)\right]^2\ \frac{dG(l)}{n} - E\left[\omega\right]^2 \\
&= \frac{\kappa}{n}(1-\frac{\kappa}{n})(\omega_0 + a\bar{l}_1)(\omega_0 + a\bar{l}_1 - 2b\mu) \\
&+ \frac{\kappa}{n}b^2(\int_{\bar{l}_1}^{v_2^*-3A^2/8} l^2\frac{dG(l)}{\kappa} - \frac{\kappa}{n}(\mu)^2) \\
&= \frac{\kappa}{n}(1-\frac{\kappa}{n})(\omega_0 + a\bar{l}_1)(E\left[\omega\right] - b\mu) \\
&+ \frac{\kappa}{n}b^2(\sigma^2 - (1-\frac{\kappa}{n})\mu^2).
\end{aligned}$$

Because the degree of ownership ω is positive only if the type 1 is liquidity constrained ($l < v_2^* - 3A^2/8$), the degree of heterogeneity of ownership will depend on the distribution among these constrained type 1 agents. If all 1's are constrained then, $\kappa = n$ and we have as in the lemma, $Var(\omega) = b^2(\int_{\bar{l}_1}^{v_2^*-3A^2/8} l^2\, dG(l)/n - \mu^2) = b^2\sigma^2$.

B.3 Proof of Proposition 6

(i) It is immediate from Lemma 5 that, if $\kappa^G = \kappa^H$, then $\int \omega(v_2^{*H}, l)dH(l) < \int \omega(v_2^{*G}, l)dG(l)$ if and only if $a\bar{l}_1^H - b\mu^H < a\bar{l}_1^G - b\mu^G$ or if $(1-\alpha)(\bar{l}_1^H - \bar{l}_1^G) < \mu^H - \mu^G$, since $a/b = 1 - \alpha$.

(ii) If $\kappa^G = \kappa^H = n$, the result is immediate from Lemma 5(ii).

B.4 Proof of Proposition 7

It is enough to provide an example. Suppose that liquidity is uniformly distributed on $[0, x]$, where $x < 3A^2/8$, and suppose that $n = 1 - \epsilon$; then $\bar{l}_1^G = \epsilon x$ and the inframarginal mean liquidity is $\mu^G = x(1+\epsilon)/2$. Suppose that all agents with liquidity in $[0, \delta]$, where $0 < \epsilon x < \delta \leq x$, have a liquidity shock and their new liquidity is δ while other 1's have the same liquidity as before. Then the new liquidity distribution is $H(l) = 0$ for $l < \delta$ and $H(l) = l/x$ for $x \geq l \geq \delta$.

The new marginal liquidity $\bar{l}_1^H$ is δ, and $\mu^H = (x^2 + \delta^2 - 2\epsilon\delta x)/(2x)$. The condition in Proposition 6 is violated when $(1-\alpha)(\delta - \epsilon x) > (\delta^2 - 2\epsilon\delta x - \epsilon x^2)/(2x)$. In particular, if $\alpha < 1/2$ then integration increases even when every 1 is given liquidity x.

B.5 Proof of Proposition 8

Let

$$\pi : [0,1] \to [0,1]$$

$$\pi(i) \geq \pi(\hat{\imath}) \Longleftrightarrow W(i) \geq W(\hat{\imath}),$$

be a reordering of the indexes of type-1 managers that is consistent with the reordering on willingness to pay induced by the shock. The marginal type-1 agent is i_π such that the Lebesgue measure of the set $\{i : W(i) \geq W(i_\pi)\}$ is n and the set of equilibrium firms is $F = \{i : \pi(i) \geq \pi(i_\pi)\}$. Let $v_2^*(A)$ be the equilibrium price in the initial situation and let $v_2^*(\hat{A})$ be the equilibrium price after the shock to the technology available to agents in $[i_0, i_1]$.

(i) (Inframarginal shocks) If $i_0 > 1 - n$, then the shocked firms become less integrated and the unshocked firms remain unaffected. This is a direct consequence of Lemma 2 and the observation that $i_\pi = 1 - n$ *and* $W(i_\pi) = v_2^*(A)$.

(ii) (Marginal shocks) If $1 - n \in (i_0, i_1)$ and $1 - n$ is still the marginal type-1 agent, then the equilibrium price increases and all firms – shocked and unshocked – integrate more. Note that $1 - n$ is still the marginal type if and only if $W(1 - n) \leq \lim_{\varepsilon \downarrow 0} W(i_1 + \varepsilon)$, for in this case all agents $i > 1 - n$ have higher willingness to pay than $1 - n$. By (8), $v_2^*(A) = W(1 - n)$ is increasing in A. Hence $v_2^*(\hat{A}) > v_2^*(A)$ and it follows that all unshocked firms $[i_1, 1]$ integrate more.

If the firm $1-n$ did not integrate before the shock (i.e., if it chose $\omega = 0$),then all $i > 1 - n$ firms also choose not to integrate because ω is decreasing in the liquidity of type 1. It is immediate that an increase in A can only lead to more integration.

Consider now the case where firm $1 - n$ integrated before by choosing a contract with $\omega > 0$. If i_1 initially chose a contract $\omega = 0$ then there exists a $k \in (1-n, i_1)$ such that all firms with $i < k$ integrate ($\omega > 0$) and all firms with $i \geq k$ do not integrate; firms with $i \geq k$ will necessarily integrate more after the shock. We have $v_2^*(A) = W(1 - n; A)$ and $v_2^*(\hat{A}) = W(1 - n; \hat{A})$. From (7) and (8) it follows that, for all shocked firms $i \in [1-n, k)$, the difference in the degree of integration before and after the shock is

$$\frac{3\hat{A}^2 - 4\bar{l}_1}{(2-\hat{A})^2} - \frac{3A^2 - 4\bar{l}_1}{(2-A)^2} > 0$$

(here $\bar{l}_1 = l_1(1-n)$) and all firms integrate more as claimed.

(iii) *(Uniform shocks) If there is a uniform shock to the technology* ($i_0 = 0$, $i_1 = 1$) *then each firm integrates more.* If $i_0 = 0$ and $i_1 = 1$, the arguments for (ii) apply because $1-n$ is still the marginal type-1 manager.

This concludes the proof of the proposition in the text. The proposition itself is incomplete, since we assume in case (ii) that the marginal type is still $1-n$. Below we consider the two cases for which the marginal agent is not $1-n$ after the shock.

Case 1: A first possibility is $i_1 < 1-n$; that is, shocked firms were not matched in the initial economy but, because $W(i_1) > v_2^*(A)$, some of these firms will be matched. In this case, entering firms are those with $i \in [i_\pi, i_1]$ while old firms are those with index $i \geq k$, where $k \geq 1-n$ satisfies $i_1 - i_\pi = k - (1-n)$ (hence firms $i \in [i_\pi, i_1]$ firms $i \in [1-n, k]$). Since $W(i_\pi) > v_2^*(A)$, the degree of integration in old firms increases. For new firms, the question is whether the increase in price $W(i_\pi) - W(1-n)$ is large enough to overcome the internal effect of shocks that push toward less integration.

Case 2: Another possibility is $1-n \in (i_0, i_1)$ and $W(1-n) > \lim_{\varepsilon \downarrow 0} W(i_1 + \varepsilon)$. Then there exists a $k > i_1$ such that $W(k) = W(1-n)$, and either $i_\pi \in (i_1, k]$ or $i_\pi \in [i_0, 1-n)$. In either case, if $l_1(i_\pi)$ is low enough then the increase in equilibrium surplus to the 2's may be small enough that the internal effect dominates and so shocked firms integrate less.

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