Name: _____

Dimensional Analysis Worksheet

In this worksheet "examples" have step-by-step solutions while "problems" only have answers. We have left you room to try all the examples before the solution is shown. We highly suggest trying to work through the examples before looking at the solutions.

Units and conversions

One of the reasons scientists prefer the metric system has to do with the ease of conversion between units. This is apparent if you consider the difference between the distance units of yards and miles versus meters and kilometers. A yard and a meter (or a mile and a kilometer) are roughly equivalent (1 yard = 0.914 meters; 1 mile = 1.61 kilometers). But consider the difference between the following examples:

Example 1. Sally walked 2.45 miles. How far did she walk in yards?

Example 2. Sally walked 3.95 kilometers. How far did she walk in meters?

These values correspond to the same distance. However, there are 1000 meters in a kilometer, but 1760 yards in a mile. In the case of Example 1, we must multiply 2.45 miles by 1760 yards to arrive at the answer:

$$\frac{2.45 \text{ miles}}{1} \times \frac{1760 \text{ yards}}{1 \text{ mile}} = 4320 \text{ yards}$$

Most people would not be able to do this calculation in their head quickly. Example 2, however, is much easier to solve, as one only has to multiply 3.95 kilometers by 1000 meters:

$$\frac{3.95 \text{ kilometers}}{1} \times \frac{1000 \text{ meters}}{1 \text{ kilometer}} = 3950 \text{ meters}$$

Powers of ten and scientific notation

The metric system is a measurement system that uses **powers of 10**. A positive power of 10 represents 10 multiplied by itself a certain number of times (10^n, where n is a positive integer). For instance, $10^3 = (10)(10)(10)$. Negative exponents mean inverse. So 10^{-n} is 1 divided by 10^n. For example:

$$10^{-5} = \frac{1}{10^5} = \frac{1}{10\times10\times10\times10\times10}$$

In the metric system, prefixes indicate powers of 10. Take the unit kilometer: the prefix "kilo" indicates that the base unit (in this case "meter") is multiplied by 1000, or 10^3. The prefix "centi", on the other hand, indicates that the base unit should be multiplied by 10^{-2} (this is equivalent to dividing by 100).

The following table indicates some of the more common prefixes:

Pico	p	0.000000000001	10^{-12}
Nano	n	0.000000001	10^{-9}
Micro	μ	0.000001	10^{-6}
Milli	m	0.001	10^{-3}
Centi	c	0.01	10^{-2}
Deci	d	0.1	10^{-1}
		1	10^{0}
Hecto	h	100	10^{2}
Kilo	k	1,000	10^{3}
Mega	M	1,000,000	10^{6}
Giga	G	1,000,000,000	10^{9}
Tera	T	1,000,000,000,000	10^{12}

Converting between units and dimensional analysis

Converting a measurement from a unit containing a prefix to the base unit is straightforward. All one has to do is multiply the given value by the power of ten indicated by the prefix.

Example 3. Convert 15 ng to g.

"n" stands for "nano," which corresponds to 10^{-9}. Replace the "n" with 10^{-9}:

$$15 \text{ ng} \rightarrow 15\ (10^{-9})\text{g}$$

and multiply:

$$15 \times 10^{-9}\text{ g} = 1.5 \times 10^{-8}\text{ g}$$

Problem 1. Convert 4.89 Ts to s.

To go in the other direction (e.g., g to ng), or to convert between prefixes (e.g., km to cm), is a little less straightforward. In these cases you will need to use fractions.

Example 4. Convert 23 g to mg.

Since one milligram is equal to 10^{-3} g = (1/1000) g, that means there are 10^3 mg in one gram:

$$1 \text{ mg} = 10^{-3} \text{ g}$$

Divide both sides by 10^{-3} (which is the same as multiplying by 1000) and simplify:

$$\frac{1 \text{ mg}}{10^{-3}} = \frac{10^{-3}\text{g}}{10^{-3}}$$

$$10^3 \text{ mg} = 1 \text{ g}$$

Because 1000 mg and 1 g are equal, you can multiply 23 g by 1000 mg/1g, since this is the same as multiplying by 1:

$$\frac{23 \text{ g}}{1} \times \frac{1000 \text{ mg}}{1 \text{ g}}$$

Because there is a "g" in both the numerator and denominator, they cancel out, and we are left with our desired units (mg):

$$\frac{23 \text{ g}}{1} \times \frac{1000 \text{ mg}}{1\text{g}} = 23{,}000 \text{ mg} = 2.3 \times 10^4 \text{ mg}$$

Note that we could have also set it up the following way (because 1 mg = 10^{-3} g):

$$\frac{23 \text{ g}}{1} \times \frac{1 \text{ mg}}{10^{-3} \text{ g}} = 23{,}000 \text{ mg} = 2.3 \times 10^4 \text{ mg}$$

In both of these cases, we were able to cancel one of the units (g) because it was in both the numerator and denominator. If we had set it up the following way:

$$\frac{23 \text{ g}}{1} \times \frac{1 \text{ g}}{1000 \text{ mg}} = \frac{23 \text{ g}^2}{1000 \text{ mg}} \quad \text{oops…}$$

we would not be able to simplify the units. When converting between units using fractions, be sure that your fractions are set up so that all units except for the desired unit cancel out (i.e., are present in both the numerator and denominator).

This process can be completed as many times as necessary to arrive at an answer, or daisy-chained in one step.

Problem 2. Convert 130 nm to km (*answer:* 1.3×10^{-10} km)

When converting between units, always take a moment to consider whether your answer is reasonable. Nanometers are *much* smaller than kilometers, so the final answer should be *much* smaller than the initial value. Another name for this method of converting between quantities is called **dimensional analysis**. Dimensional analysis is a method of problem solving that allows us to use relationships between quantities as "stepping stones" to solving complicated problems.

Quantities. There are two types of quantities used in dimensional analysis:

1) An intrinsic quantity (e.g., 5 kilometers)
2) A relationship, either given or known (e.g., 24 hours per day, 10 pens per box)

Proportions. Relationships can be turned into proportions, because:

24 hours = 1 day

Divide both sides by 1 day

$$\left(\frac{24 \text{ hours}}{1 \text{ day}}\right) = \left(\frac{1 \text{ day}}{1 \text{ day}}\right) = 1$$

If you can compose a sentence containing "per" or "in a", you can compose a proportion. As we saw above, proportions can be multiplied together, and some units cancelled, because of the associative property:

$$\left(\frac{a}{b}\right)\left(\frac{c}{d}\right) = \left(\frac{a}{d}\right)\left(\frac{c}{b}\right)$$

For example:

$$\frac{35 \text{ miles}}{1 \text{ gallon}} \times \frac{12 \text{ gallon}}{\text{tank}} = \frac{35 \times 12 \text{ miles}}{1 \text{ tank}} \times \frac{\text{gallon}}{\text{gallon}} = 420 \text{ miles per tank}$$

Inversion of conversion factors. Because proportions are essentially equal to one, they can also be flipped to cancel units with no ill effect.

Example 5. How many days are equal to the length of one 50-minute lecture?

Solution: let's start with the fact that lectures are 50 minutes and work from there:

$$\left(\frac{50\text{ min}}{1\text{ lecture}}\right)\left(\frac{60\text{ min}}{1\text{ hour}}\right)\left(\frac{24\text{ hours}}{1\text{ day}}\right) = 72000\ \frac{\text{min}^2}{\text{lecture}\ \cdot \text{day}}$$

These units don't make sense! We need to flip some conversion factors to cancel units, not compound them.

$$\left(\frac{50\text{ min}}{1\text{ lecture}}\right)\left(\frac{1\text{ hour}}{60\text{ min}}\right)\left(\frac{1\text{ day}}{24\text{ hours}}\right) = 0.03\ \frac{\text{days}}{\text{lecture}}$$

Problem 3. The speed of light is 3.0×10^8 m/s. The Earth's circumference at the equator is 40,000 km. How long (in theory) would it take in seconds for a laser beam to travel around the equator? (*answer: 0.13 s)*

<u>Practice Problems (by approximate level of difficulty)</u>

Straightforward

1. How many seconds are in a year? (Answer: 3×10^7 s)

2. The circumference of Earth is 25,000 miles. What is the circumference in cm? (Answer: 4.0×10^9 cm)

3. To reach the recommended daily intake of calcium, a person must drink 600. mL of milk a day. If 200. mL of milk contains 300. mg of calcium, how much calcium, in kg, is a person recommended to intake in 30. days? (Answer: 0.027 kg)

4. There are 2600. miles between Boston and Los Angeles (in a straight line). If a plane flies at 600. miles/hour. How long is the flight between Boston and LA? (Answer: 4.33 hours)

5. If you earned one penny for every 10. seconds of your life then how many dollars would you have after 65 years? (Answer: \$ 2.0×10^6)

Elementary

6. Levoxyl is a drug used to treat hypothyroidism. If a patient takes one 75.0 ng tablet per day, how many milligrams of Levoxyl are in their 1 month (30. day) supply? (Answer: 2.3×10^{-3} mg/month)

7. A car's gas tank holds 12 gallons and is ¼ full. The car gets 20. miles/gallon. You see a sign saying "Next gas 82 miles". Can you make it to the gas station before running out of gas? (Answer: no)

8. A popular web site states that a 200. pound person will burn 450. calories/hour bicycling (moderate effort) and 650. calories/hour rock climbing (ascending). A 150. pound person decides to start a training routine in which she will bicycle for 45 minutes, 3 times a week and rock climb for 1.50 hours every weekend. How many calories will she have burned after 8 weeks of training (assuming she consumes at the same rate)? What percentage of those are from bicycling? (Answer: 15,900 calories, 50.9 %) (Question: why don't "3 times" and "8 weeks" limit the significant figures in the problem?)

9. Light travels at 2.9979 x 10^{10} cm/s. How far would light travel in a year? (Answer: 9.454 x 10^{17} cm)

10. The human body is 60. % water by mass. How many grams of water are there in a person who weighs 90. kg? (Answer: 54 kg)

11. An average man requires about 2.0 mg of riboflavin (Vitamin B12) per day. One slice of cheese contains 5.5 µg of riboflavin. How many slices of cheese would a man have to eat per day if it were his only source of riboflavin? (Answer: 360 slices)

Moderate

12. A diabetic is recommended to use 1 cm^3 of insulin for every 10. g of carbohydrates consumed. The recommended daily intake of carboydrates is 300. g. A diabetic has eaten a slice of toast and has consumed 5.0 % of their daily value of carbohydrates. How many mL of insulin should the diabetic use to maintain a proper blood sugar level after eating the piece of toast? (Answer: 1.5 mL)

13. Every 3 hours a fast food employee wraps 350 hamburgers. He works 8 hours per day, 5 days a week. He gets paid $700. for every 2 weeks. How many hamburgers will he have to wrap to make his first one million dollars? (Answer: 1.3 x 10^7 burgers)

14. The roof of a building is 0.20 km^2. During a rainstorm, 5.5 cm of rain was measured to be sitting on the roof. What is the mass of the water on the roof after the rainstorm? (Answer: 1.1 x 10^{10} g)

15. The ideal gas constant was experimentally found to be 8.3 J/(K mol). At a temperature of 300. K, how much energy (in J) would a sample of 48 x 10^{30} atoms of water contain? (Answer: 1.9 x 10^{11} J)

16. The density of silver is 10.50 g/cm^3. If 5.50 g of pure silver pellets are added to a graduated cylinder containing 11.0 mL of water, to what volume will the water in the cylinder rise? (Answer: 11.5 mL)

17. The re-entry speed of the Apollo 10 space capsule was 11.0 km/s. How many hours would it have taken for the capsule to fall through the 25,000 miles of stratosphere? (Answer: 1.0 hour)

Difficult

18. Convert 10.0 g cm^3/s^2 to kg m^3/hr^2 (Answer: 0.130 kg m^3/hr^2)

19. The bromine content of the ocean is about 65 g of bromine per million g of sea water. How many cubic meters of ocean must be processed to recover 500. mg of bromine, if the density of sea water is 1.0x10^3 kg/m^3? (Answer: 7.7×10^{-3} m^3)

20. If 20.0 g of coal are burned, heating 1.00 L of water, how much hotter will the water get? Assume all of the heat lost by the coal is gained by the water. (Answer: 129 ^oC) Additional information: Density of water, 1.00 g/mL; specific heat of water, 4.184 J/(g·°C); density of coal, 1506 kg/m^3; heat of combustion of coal, 27.00 MJ/kg.

21. Diamonds are measured in carats and 1 carat = 0.200 g. The density of diamond is 3.51 g/cm^3. A diamond is dropped into a graduated cylinder filled with 30 mL of water and the final volume is measured to be 35 mL. How many carats is the diamond? (Answer: 88 carats)

22. I prefer my coffee with sugar in it. Generally, I add 0.5 g of sugar ($C_{12}H_{22}O_{11}$) to my thermos of coffee. If I were to buy a new thermos that was 1.5 times the size of my current thermos, how much additional sugar would I need to add to maintain the same level of sweetness in my coffee? (Answer: 0.75 g)

