14. Particle in a Box: Standing Waves

Wave properties of an electron result in discrete energy levels in an atom. For classical standing waves, quantized energy values can also be seen, such as the movement of a spring or a jump rope held at two ends. This is an appropriate analogy to an electron inside an atom because it acts as a standing wave that is bound much like a spring or jump rope. These boundaries are energy barriers such that the electron’s energy is much lower than the barriers, preventing it from escaping.

To simplify this phenomenon, a one-dimensional electron wave can be trapped in a “box” of energy barriers. The result is a one-dimensional standing wave, much like the analogy to a spring fixed at both ends. Understanding the different energy states of these standing waves gives insight into the quantized properties of electrons (see http://goo.gl/bqbJs7 pp. 24 - 27 for more details).

Go to the following applet:
http://quantum.bu.edu/CDF/101/12-ParticleInABox-1.cdf

When this first opens, you will see two sliders at the top. The first, labeled $L$, indicates the separation between the two barriers of the box. We will call this the size of the box. The second, labeled Loops, indicates the number of loops in the electron wave.

1) Set $L = 5$ nm. Draw the electron wave for each of the following number of loops ($n$).

$n = 1$  

$n = 2$

$n = 3$  

$n = 4$

$n = 7$  

$n = 10$

2) What does the number of loops ($n$) refer to in the electron wave? (Hint: Think about wavelength)
3) Will you ever have only part of a loop in the box? What does this mean for the number of de Broglie wavelengths you can have inside the box?

4) Set $n = 1$. Change $L$ and observe what happens to the electron wave.
   a) What happens to the amplitude of the wave as you change the size of the box?
   b) What happens to the de Broglie wavelength as you change the size of the box?

5) Now that you know that you must have only whole numbers of loops in the box (contained in an atom), what does it mean to say that the electron energy is quantized? (Hint: Remember the relationship between wavelength and energy).