13. Particle in a Box: de Broglie Wavelength

A Historical Introduction

The behaviors of light and matter may strike you as somewhat odd. For example, why would light and matter only interact at certain resonant frequencies? Why does an atom have distinct energy levels? Why is there such a big difference between the classical and quantized world? The first glimpse at answering these questions came from a set of experiments conducted between 1921 and 1927. In these experiments, electrons were passed through thin crystals and observed on the other side. If the electron were a classical particle, the pattern on the other side should have looked like one spot of passing electrons. What they saw, however, was a diffraction pattern similar to the diffraction of light. The only way this pattern could be created was if electrons had wave-like properties like those of light. One of the greatest implications of these findings was that the mass of an electron is spread out energetically in a wave, not localized in one point like a classical object (see http://goo.gl/bqbJs7 pp. 24-40 for more details). This is why we say that atoms have “electron clouds”.

Observing the wave-like properties of electrons is the key to the quantized nature of light and matter interactions. Let’s take a look at how a classical example of standing waves gives us information about the quantized nature of matter.

The images below show an example of a classic particle (a baseball) that is localized and a classic wave (sound) that is delocalized. Answer the following questions about these diagrams.

1) Is it possible to circle the exact location of the baseball in this diagram (i.e. is it localized)? Why or why not?

2) Is it possible to circle the exact location of the sound in this diagram (i.e. is it localized)? Why or why not?

3) In your own words, based on the answers to (1) and (2), define, “delocalization”.

Remember, analogies are a good way to understand abstract topics, but they are limited. The analogies above do not give exact representations of electrons, but instead help to explain some of their more abstract behavior.
The de Broglie Wavelength

The properties of light we have studied leading up to its interaction with matter are wavelength, frequency, and energy. For the electron, de Broglie determined that the wavelength must be calculated using both the delocalized wave properties, the mass and velocity of the electron itself. The result was the following equation:

\[ \lambda_{\text{de Broglie}} = \frac{h}{p} \]  

Where \( \lambda_{\text{de Broglie}} \) is the wavelength of the electron, \( h \) is Planck’s constant (6.626 x 10\(^{-34}\) m\(^2\)kg/s), and \( p \) is the electron’s momentum (\( p = mv \)). Using this equation, answer the following questions.

1) Do classical objects, such as a thrown baseball, have a de Broglie wavelength? Why or Why not?

2) What is the de Broglie wavelength for an electron traveling at 2.2 x 10\(^6\) m/s (The mass of an electron is 9.11 x 10\(^{-31}\) kg).